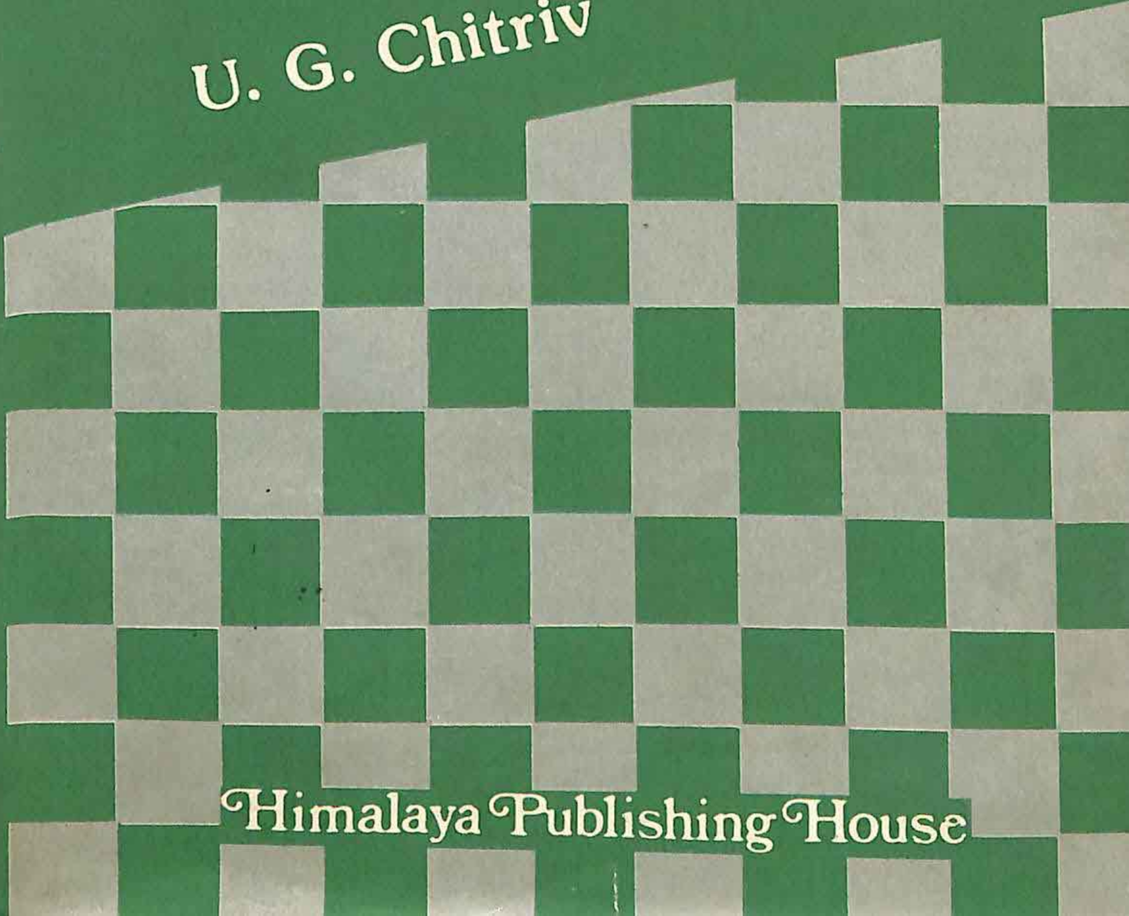
AUSUBEL Vs. BRUNER MODEL FOR TEACHING MATHEMATICS

U. G. Chitriv



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ABOUT THE BOOK

The present book is an abridged version of the author's doctoral dissertation submitted to the Nagpur University in 1983. It is a novel work in the field of mathematics education. It is also a premier thesis in India involving two important models of teaching in the information processing family developed by Bruce Joyce and Marsha Weil, namely, Ausubel's Advance Organizer Model and Bruner's Concept Atttainment Model.

The researcher has compared the two models on the various criteria of concept -acquisition in mathematics and brought forth a few valuable conclusions. The research is characterised by discovery of new facts and bears ample evidence of researcher's profound scholarship, his capacity for critical thinking and sound judgement. The literary presentation is superb and evinces an organized mind behind it.

In view of fresh emphasis on mathematics education envisaged in the new National Policy of Education, 1986 and the rigorous efforts by the National Council of Educational Research and Training to orient teachers in Models of Teaching, this book should serve as a valuable reference to all teachers, teacher-educators, educational researchers, resource persons and educationists in India and abroad.

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U.G. CHITRIV
M.Sc. (Maths.), M. Ed. Ph. D.
Post-Graduate Department of Education
Nagpur University, Nagpur

S.C.E.R.T., West Be
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Calcutta
25/3, B. C. Road

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New Delhi-110 002
Phone: 270392

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PREFACE

In the year 1979, the Nagpur University started a new Post-Graduate Course in Mathematics Education and the responsibility of imparting instruction in "Models of Teaching" fell on my shoulders. In order to provide adequate and up-to-date information about these models, I started scanning the related literature. Perusal of periodicals, in particular, revealed to me that there has been widespread acceptance of utility of these models for classroom teaching notwithstanding the fact that their efficacy for the attainment of specific educational objectives has not been supported by sufficient research evidence.

Ausubel's 'Advance Organizer Model' and Bruner's 'Concept Attainment Model' appeared to have attracted greater attention of educationists in India and the world at large. The former espoused an expository approach and the latter a discovery approach to teaching. Supporters of these two models were, thus, inevitably drawn into two antagonistic positions reminiscent of long standing expository-discovery dilemma.

Later experience at supervising the lessons of M.M.Ed. (Master of Mathematics Education) students planned as per tenets of these two models made me increasingly aware of their potentialities for teaching mathematical concepts. At the same time, I visualized that, by virtue of their well defined syntaxes, these models could serve as unanimous representatives of expository and discovery methods of teaching.

I, therefore, contemplated that by subjecting these two models to empirical tests under comparable conditions their relative effectiveness for the acquisition of mathematical concepts by Indian students could be ascertained and, in addition, some new light could be thrown on the expository-discovery controversy which still remained unresolved.

This knowledge was obviously essential in order that we should have some empirically based justification for the recommendation of these models for day-to-day teaching of mathematical concepts in our schools and colleges. The motive for the present study sprang from this upright demand of mathematics education in the Indian subcontinent.

The task was monumental and the resources meagre. But constant inspiration and encouragement of very many friends both in India and abroad provided me the strength and spirit to overcome the hurdles and bring this study to successful completion.

The study has been provided into six chapters. The first chapter outlines theoretical frame of the study and states its purpose and objectives. The second chapter has been devoted to review of related research and formulation of hypotheses tested in this study. The third chapter presents blueprint of the study and describes the procedures adopted in its conduct. Statistical analyses of the data have been presented and results from each analysis brought out in the fourth chapter. In order to point out agreement and disagreement of this study with those conducted in the recent past, the results have been discussed at length in the fifth chapter. The sixth chapter summarizes the whole study and states its major findings and conclusions.

I owe a debt of gratitude to Dr. Suryakant Jha, my supervisor, who taught me what it is to be a research worker. The penetrating questions he asked, implicit guidance he offered, and unfailing encouragement he gave, were instrumental in streamlining the present study.

This research work was pursued at the Centre of Advanced Study in Education, M.S. University of Baroda, under the award of Teacher Fellowship by the University Grants Commission, New Delhi, during the academic session 1981-82. I am thankful to the Centre and the Commission for the award.

The Indian Council of Social Science Research, New Delhi, referred this thesis for evaluation to Prof. M.B. Buch, Chairman, Society for Educational Research and Development, Baroda, and granted financial assistance for its publication on his recommendation. I am grateful to the Council and their consultant for the kind gesture.

Himalaya Publishing House, Bombay, has efficiently shouldered the responsibility of publishing the thesis within a short span of time. It would be my pleasure to remain indebted to them for ever.

The study reflects an honest endeavour to add a drop to the ocean of knowledge by whatever limited power and potentialities I might have possessed. It is by no means an end in itself, but rather a means towards the end. If research workers in the field of mathematics education come forward to share some fresh challenges this study has given me realization of, responsibility would rest less heavily on me.

U.G. Chitriv

CONTENTS

Preface (*vi*)
List of Tables (*xvii*)
List of Figures (*xxi*)
List of Appendices (*xxiii*)

Chapter I. Introduction **1—31**
1.0.0. Context of the Problem 1
1.1.0. Mathematics Teaching 4
1.1.1. The Nature of Mathematics 4
1.1.2. The Nature of Teaching 6
1.1.3. Central Objective of Mathematics Teaching 8
1.2.0. Theory, Model, Strategy and Method 8
1.2.1. Theories of Teaching 8
1.2.2. Models of Teaching 9
1.2.2.1. Concept Attainment Model 11
1.2.2.2. Advance Organizer Model 15
1.2.3. Strategies of Teaching 18
1.2.4. Methods of Teaching 20
1.3.0. Cognitive Entry Behaviours 21
1.3.1. Cognitive Style 22
1.4.0. Research on Teaching Methods 24
1.5.0. Statement of the Problem 26
1.6.0. Significance of the Study 26
1.7.0. Objectives of the Study 27
1.8.0. Assumptions Underlying the Study 28
1.9.0. Scope and Delimitation of the Study 29
1.10.0. Definitions of the Terms Used 30

Chapter II. Review of Research 32—68
2.0.0. Prologue 32
2.1.0. Classification of Studies 33
2.2.0. Expository-Discovery Studies in Laboratory Conditions 34
2.2.1. An Overview of Laboratory Studies 35
2.3.0. Expository-Discovery Studies in Classroom Situations 35
2.3.1. An Overview of Classroom Studies 41
2.4.0. The Studies Involving Cognitive Style Dimension 42
2.4.1. An Overview of Cognitive Style Studies 46
2.5.0. The Studies Involving Advance Organizers 47
2.5.1. An Overview of Ausubelian Studies 50
2.6.0. The Studies Involving Exemplars-Non-exemplars 51
2.6.1. An Overview of Brunerian Studies 55
2.7.0. Expository-Discovery Studies Involving Some Elements of Ausubel and Bruner Models 56
2.7.1. An Overview of Ausubel Vs. Bruner Studies 58
2.8.0. Implications of Previous Research for the Present Study 59
2.9.0. Rationale for the Study 61
2.10.0. Hypotheses 63
Chapter III. Design and Procedure 69—98
3.0.0. Prologue 69
3.1.0. Experimental Variables 70
3.1.1. Independent Variables 71
3.1.2. Dependent Variables 72
3.1.3. Moderator Variable 72

3.1.4. Control Variables 72
3.2.0. Extraneous Variables and their Control 73
3.2.1. Controls for Internal Validity 73
3.2.2. Controls for External Validity 74
3.3.0. Experimental Design 74
3.4.0. Selection of Subjects 76
3.4.1. Selection of School Classes 76
3.4.2. Allotment of Treatments 78
3.4.3. Final Sample 78
3.5.0. Choice of Content 79
3.6.0. Lesson Plans 80
3.6.1. Format of Ausubel Lesson Plan 80
3.6.2. Format of Bruner Lesson Plan 81
3.6.3. Format of Traditional Lesson Plan 81
3.7.0. Instrumentation 81
3.7.1. Raven's Standard Progressive Matrices 82
3.7.2. Cognitive Style Test 83
3.7.3. Previous Knowledge Test 84
3.7.4. Concept Knowledge Test 84
3.7.5. Concept Transfer Test 85
3.7.6. Heuristic Transfer Test 85
3.7.7. Concept Retention Test I 86
3.7.8. Concept Retention Test II 87
3.7.9. Observer's Rating Scale 87
3.8.0. Observer Raters 88
3.9.0. Pre-testing 89
3.10.0. The Conduct of Experiment 90
3.11.0. Post-testing 92
3.11.1. Administration of Retention Tests 93
3.12.0. Scoring 93
3.13.0. Inter Observer Reliability 94
3.14.0. Statistical Techniques for Data-Analysis 95
3.14.1. Levels of Significance 98

Chapter IV. Analyses and Results 99—144
4.0.0. Prologue 99
4.1.0. Assessment of Teacher Fidelity 100
4.2.0. Identification of Cognitive Style Subgroups 102
4.3.0. Descriptive Analysis of the Scores on Intelligence Test 103
4.4.0. Descriptive Analysis of the Scores on Previous Knowledge Test 103
4.5.0. Descriptive Analysis of the Scores on Concept Knowledge Test (CKT) 104
4.6.0. Inferential Analysis of the Scores on Concept Knowledge Test for the Whole Groups, A, B, C 104
4.6.1. Inferential Analysis of the Scores on Concept Knowledge Test for Categorical Style Subgroups of the Groups A, B, C 107
4.6.2. Inferential Analysis of the Scores on Concept Knowledge Test for Descriptive Style Subgroups of the Groups A, B, C 109
4.6.3. Inferential Analysis of the Scores on Concept Knowledge Test for Relational Style Subgroups of the Groups A, B, C 109
4.7.0. Descriptive Analysis of the Scores on Concept Transfer Test (CTT) 111
4.8.0. Inferential Analysis of the Scores on Concept Transfer Test for the Whole Groups A, B, C 111
4.8.1. Inferential Analysis of the Scores on Concept Transfer Test for Categorical Style Subgroups of the Groups A, B, C 114
4.8.2. Inferential Analysis of the Scores on Concept Transfer Test for Descriptive Style Subgroups of the Groups A, B, C 115
4.8.3. Inferential Analysis of the Scores on Concept Transfer Test for Relational Style Subgroups of the Groups A, B, C 115

4.9.0. Descriptive Analysis of the Scores on Heuristic Transfer Test (HTT) 116

4.10.0. Inferential Analysis of the Scores on Heuristic Transfer Test for the Whole Groups A, B, C 116

4.10.1. Inferential Analysis of the Scores on Heuristic Transfer Test for Categorical Style Subgroups of the Groups A, B, C 119

4.10.2. Inferential Analysis of the Scores on Heuristic Transfer Test for Descriptive Style Subgroups of the Groups A, B, C 120

4.10.3. Inferential Analysis of the Scores on Heuristic Transfer Test for Relational Style Subgroups of the Groups A. B, C 122

4.11.0. Descriptive Analysis of the Scores on Short-Term Retention Test 124

4.12.0. Inferential Analysis of the Scores on Short-Term Retention Test for the Whole Groups A, B, C 124

4.12.1. Inferential Analysis of the Scores on Short-Term Retention Test for Categorical Style Subgroups of the Groups A, B, C 127

4.12.2. Inferential Analysis of the Scores on Short-Term Retention Test for Descriptive Style Subgroups of the Groups A, B, C 129

4.12.3. Inferential Analysis of the Scores on Short-Term Retention Test for Relational Style Subgroups of the Groups A, B, C 131

4.13.0. Descriptive Analysis of the Scores on Long-Term Retention Test 133

4.14.0. Inferential Analysis of the Scores on Long-Term Retention Test for the Whole Graops A, B, C 133

4.14.1. Inferential Analysis of the Scores on Long-Term Retention Test for Categorical Style Subgroups of the Groups A, B, C 135

4.14.2. Inferential Analysis of the Scores on Long-Term Retention Test for Descriptive Style Subgroups of the Groups A, B, C 137

4.14.3 Inferential Analysis of the Scores on Long-Term Retention Test for Relational Style Subgroups of the Groups A, B, C 138

4.15.0. Panorama of Analyses 140

4.16.0. Bird's Eye-view of Results 140

Chapter V. Discussion and Implications **145—154**

5.0.0. Prologue 145

5.1.0. Efficacy of Ausubel Strategy 145

5.2.0. Efficacy of Bruner Strategy 146

5.3.0. Relative Effectiveness of Ausubel and Bruner Strategies 147

5.3.1. Searchlight on Expository-Discovery Dilemma 148

5.4.0. Effect of Conceptual Style Preference 150

5.4.1. Impact of Categorical Style Preference 150

5.4.2. Impact of Descriptive Style Preference 151

5.4.3. Impact of Relational Style Preference 151

5.4.4. Differential Effects of the Three Conceptual Style Preferences 151

5.5.0. Implications for Teaching 152

5.6.0. Suggestions for Future Research 153

Chapter VI. Summary and Conclusions **155—164**

6.0.0. The Problem 155

6.1.0. Objectives 156

6.2.0. Sample 157

6.3.0. Material 157
6.4.0. Experimental Design 158
6.5.0. The Experiment 158
6.6.0. Analyses 159
6.7.0. Results 161
6.8.0. Conclusions 162
6.9.0. Implications 163
6.10.0. Suggestions 163

Bibliography **165—188**

Appendices **189—224**

LIST OF TABLES

Table	*Title*	*Pages*
3.1.	Summary of the Data for Preliminary Selection of Schools	77
3.2.	Summary of Preliminary Data for the Three Schools Selected for Experimental Teaching	79
3.3.	Summary of Preliminary Data for the Final Sample of Subjects	79
3.4.	Inter Observer Reliability for Each Pair of Observers, Observing the Lessons of the Three Strategies	96
4.1.	Grand Scores for the Three Strategies Obtained on the Basis of Observer Raters' Reports	100
4.2.	Significance of Differences Between the Grand Score and Perfect Score for the Three Strategies	101
4.3.	Significance of Differences Between the Grand Scores of the Three Strategies	102
4.4.	Cognitive Style-wise Classification of Students in Groups A, B, C	102
4.5.	Means and Standard Deviations of the Three Groups and Their Style-wise Subgroups on Intelligence Test (RPM)	103
4.6.	Means and Standard Deviations of the Three Groups and Their Style-wise Subgroups on Previous Knowledge Test (PKT)	104
4.7.	Means and Standard Deviations of the Three Groups and Their Style-wise Subgroups on Concept Knowledge Test (CKT)	105
4.8.	Significance of Difference Between Adjusted Means of Groups A, B, C on Concept Knowledge Test	106
4.9.	Significance of Difference Between Adjusted Means of Subgroups Ac, Bc, Cc on Concept Knowledge Test	108

Table *Title* *Pages*

4.10. Significance of Difference Between Adjusted Means of Subgroups Ar, Br, Cr on Concept Knowledge Test 110

4.11. Means and Standard Deviations of the Three Groups and Their Style-wise Subgroups on Concept Transfer Test (CTT) 112

4.12. Significance of Difference Between Adjusted Means of Groups A, B, C on Concept Transfer Test 113

4.13. Means and Standard Deviations of the Three Groups and Their Style-wise Subgroups on Heuristic Transfer Test 117

4.14. Significance of Difference Between Adjusted Means of Groups A, B, C on Heuristic Transfer Test 118

4.15. Significance of Difference Between Adjusted Means of Subgroups Ac, Bc, Cc on Heuristic Transfer Test 120

4.16. Significance of Difference Between Adjusted Means of Subgroups Ad, Bd Cd on Heuristic Transfer Test 122

4.17. Significance of Difference between Adjusted Means of Subgroups Ar, Br, Cr on Heuristic Transfer Test 124

4.18. Means and Standard Deviations of the Three Groups and their Style-wise Subgroups on Short-Term Retention Test 125

4.19. Significance of Difference Between Adjusted Means of Groups A, B, C on Short-Term Retention Test 126

4.20. Significance of Difference Between Adjusted Means of Groups Ac, Bc, Cc on Short-Term Retention Test 128

4.21. Significance of Difference between Adjusted Means of Subgroups Ad, Bd, Cd on Short-Term Retention Test 130

4.22. Significance of Difference between Adjusted Means of Subgroups Ar, Br, Cr on Short-Term Retention Test 132

Table	*Title*	*Pages*
4.23.	Means and Standard Deviations of the Three Groups and Their Style-wise Subgroups on Long-Term Retention Test	133
4.24.	Significance of Difference Between Adjusted Means of Groups A, B, C on Long-Term Retention Test	135
4.25.	Significance of Difference Between Adjusted Means of Subgroups Ac, Bc, Cc on Long-Term Retention Test	136
4.26.	Significance of Difference Between Adjusted Means of Subgroups Ar, Br, Cr on Long-Term Retention Test	139
4.27.	Review of Inferential Analyses	141
4.28.	Summary of Results	143
6.1.	Validity and Reliability Coefficients of the Tests Used	158

LIST OF FIGURES

Figure	*Title*	*Pages*
3.1.	Shulman Scheme for Examining the Variables	70
3.2.	Diagrammatic Presentation of the Non-Equivalent Control Group Design	75
3.3.	Initial Outcomes in Random Assignment of Treatments to Classes	78

LIST OF APPENDICES

Appendix	*Title*	*Pages*
A.	Hierarchy of Concepts	189
B.	Units of Teaching	190
C.	Lessson Plan of Ausubel Strategy	190
D.	Lesson Plan of Bruner Strategy	197
E.	Lesson Plan of Traditional Strategy	204
F.	Cognitive Style Test	208
G.	Previous Knowledge Test	214
H.	Concept Knowledge Test	215
I.	Concept Transfer Test	217
J.	Heuristic Transfer Test	219
K.	Concept Retention Test I	220
L.	Concept Retention Test II	221
M.	Observer's Rating Scale	222

CHAPTER I

Introduction

1.0.0. Context of the Problem

Epistemological approaches conflicted with each other ever since the days of Plato and Aristotle. Although psychology established itself as a discipline distinguishable separately from philosophy nearly a century ago, it continued to cherish competing theories of learning which were largely inherited from armchair philosophies in the area. Educators often enthusiastically employed psychological theories to design improved instructional paradigms, but fell into controversies over the question of preferable methods of teaching, obviously for certain lack of experimental support.

Mathematics educators have particularly been more adept in devising instructional strategies for the teaching of mathematical concepts on the basis of contemporary psychological thinking. But conflicting—even contrasting—views expressed by psychologists working in this field seem to have put them on the horns of a dilemma arising out of expository-discovery controversy.

Polya (1948) and Piaget (1953) advocated discovery approach for the teaching of mathematical concepts. Bruner (1961) provided strong support to their thinking and became the main contender of discovery method. According to him, "The most uniquely personal of all that he (anyone) knows is that which he has discovered for himself." He asserts that discovery is "...a necesssary condition for learning the variety of techniques of problem solving, of transforming information for better use, indeed, for learning how to go about the very task of learning". While recommending discovery approach for the teaching of mathematics, Bruner (1966, p. 72) argues that, "We teach a subject not to produce little living libraries on the subject, but rather to get the student to think mathematically for himself...".

Ausubel (1964), on the other hand, contends that, "discovery methods are no panacea, and that expository and problem solving approaches can be rote or meaningful according to the conditions under which they operate." He further argues that "...most of what he (anyone) really knows and meaningfully understands, consists of insights discovered by others, which have been communicated to him in meaningful fashion". He feels that the use of discovery method on an all-or-none basis is warranted neither by logic nor evidence. Gagne (1965) and Skemp (1971) also seem to favour expository approach for the teaching of mathematical concepts to some extent.

Extensive research has been done on the relative efficacy of expository and discovery approaches for the teaching of mathematical concepts. But, as Henderson (1963, p. 1020) aptly puts it, "One conclusion (that can be drawn from these studies) is that the evidence is not conclusive." Wittrock (1966, p. 44) identified 'semantic inconsistencies in designating the methods' as one of the reasons for equivocal research evidence in this area. Bittinger (1968, p. 78) pointed out that, "the didactic method has not been given fair chance in control group" in some of the previous studies. Gustafsson (1975) criticised many studies on this subject for their low technical quality. Wallen and Travers (1963, p. 494) and Cronbach (1966, p. 76) have gone to the extent of suggesting that this polemic question should be set aside. One would, however, be inclined to agree with Cantor (1982, p. 454) when he asserts that "The problem appears to have sufficient pedagogical and theoretical importance to justify further attempt to shed new light on the matter."

One of the probable reasons for inconclusive research evidence in this area, according to present investigator, was a piecemeal approach adopted by some of the previous researchers. Since the whole may be greater than sum of its parts, it was conjectured that an eclectic approach—covering all important facets of expository and discovery presentations—would bring forth some new evidence. This conjecture led to plan the study which would involve most inclusive forms of expository and discovery presentations. Models of Teaching, developed by Joyce and Weil (1972) from theoretical standpoints of Ausubel and Bruner, appeared to be best suited for the purpose.

Ausubel's 'Advance Organiser Model' was considered to be the choicest representative of expository method, because it emphasized organized exposition of subject-matter as hierarchy of concepts and also its meaningful assimilation in students' cognitive structure. Bruner's 'Concept Attainment Model' was regarded as best representative of discovery method, since it involved learning of subject matter by discovering concepts embedded in examples and non-examples, and, furthermore, an analysis of concept attainment strategies which could provide a training in heuristics—the crux of discovery. According to Weil and Joyce (1978, p. 21):

> Two of the models, the Concept Attainment Model and the Advance Organizer Model, are concerned with the acquisition and retention of concepts. The former involves an inductive mode of teaching, whereas the latter reflects a more didactic, expository form of instruction.

Since these models had well-defined syntaxes, it was further contemplated that the comparison between them would be relatively free from those deficiencies which possibly vitiated previous research on expository and discovery modes of presentation.

Review of research (Chapter II) revealed that relative effectiveness of the full-fledged forms of these two models, for acquisition of mathematical concepts, has not, so far, been investigated in any study, either in India or abroad. Indian educators, however, demonstrated growing awareness of the utility of these models for class-room teaching; as was later evidenced, by a series of orientation programmes, in models of teaching, for teacher educators, organised by the Department of Education, National Council of Educational Research and Training, New Delhi, throughout India. The present investigator, therefore, felt an imperative need of undertaking experimental research on this subject.

1.1.0. Mathematics Teaching

According to Howson (1973, p. 61), the working group on research in the teaching of mathematics at the Second International Congress on Mathematical Education held that the main role of research in mathematical education was the improvement of mathematics teaching. Research can conceivably contribute towards improving the teaching of mathematics broadly in two ways, *viz.* :

(1) developing theoretical models for the teaching of mathematics; and

(2) testing the existing models in classroom conditions.

Conduct of research in either of these ways has its underpinning in the objectives of mathematical instruction. Although specific objectives of instruction may change with time and place, the central objective of mathematics instruction remains relatively invariant. It emerges from the meaning of mathematics and the concept of teaching. In order to identify this central objective it would be necessary to discuss the nature of mathematics and teaching.

1.1.1. The Nature of Mathematics

Mathematics is significant for both a scholar and a layman though their perceptions may differ. A scholar may find in it the purest form of abstraction and a layman may look at it as a tool for solving numerical problems. To Polya (1948, p.VII) "Mathematics presented in the Euclidian way appears as a systematic deductive science; but mathematics in the making appears as an experimental inductive science." According to Poincare (cited by Stewart, 1965, p. 239), "A scientist worthy of the name, above all a mathematician, experiences in his work the same impression as an artist, his pleasure is as great and of the same nature." Mathematics is thus a science and also an art. As a way of thinking, it can be used to determine whether or not an idea is true or at least probably true, and as a way of reasoning it gives us insight into the power of the human mind.

The foregoing passage describes the extrinsic nature and function of mathematics. Following are some of the definitions which reveal intrinsic nature of mathematics :

Dienes (1960, pp. 10-11) : Mathematics is based on experience, it is the crystallisation of relationships into a beautifully regular structure distilled from our actual contact with the world.

Skemp (1963, p. 41) : Mathematics is not a collection of facts, which can be demonstrated and verified in the physical world, but a structure of closely related concepts, arrived at by a process of pure thought.

Isaacs (1963, p. 25) : Mathematics is virtually a product of pure thought. Even though it draws its starting points from the world of experience, it refines them into concepts well beyond the scope of that world, and then, by purely logical processes of combination, inference and construction builds the most elaborate thought scheme.

Wilder (1972, p. 41) : In the beginning, it derived its concepts, such as number, directly from the external world.... But as mathematics evolved, a higher order abstraction was achieved, in which concepts began to be applied to concepts.

Lovell (1966, p. 209) : Once the natural numbers and their properties have been isolated as it were, by more abstract thinking which is able to reflect on the results of intuition, mathematics at once becomes a more conceptualised construction.

From these definitions five deductions emerge:

1. Mathematics is a product of pure thought.
2. Mathematics is a highly conceptualised construction.
3. Mathematics is a regular structure of closely related concepts.
4. Elementary concepts of mathematics can be acquired by direct experience from the physical world.

5. Higher order abstract concepts in mathematics can be arrived at by the process of pure thought and logic or intuition.

1.1.2. The Nature of Teaching

The purpose of teaching is to maximize learning. According to Ausubel (1968, p. 12) "the facilitation of learning is the only proper end of teaching." Teaching obviously includes transmission of information and creating appropriate situations and conditions or proposing activities designed to facilitate learning.

There is a controversy whether teaching is an art or a science. Highet (1950, pp.vii-viii) writes :

> I believe that teaching is an art, not a science.......Teaching is not like inducing a chemical reaction; it is much more like painting a picture or making a piece of music or on a lower level like planting a garden or writing a friendly letter.

Gage (1964, p. 271) argues : "Although teaching requires artistry, it can be subjected to scientific scrutiny. The power to explain, predict and control that may result from such scrutiny will not dehumanize teaching." In an attempt to resolve the controversy, Gage 1978, pp. 17-20) distinguishes between a science of teaching and a scientific basis for the art of teaching in the following words :

> A science of teaching implies that good teaching will some day be attainable by closely rigorous laws that yield high predictability and control. ....a scientific basis consists of knowledge of regular non-chance relationships in the realm of events with which the practice is concerned.

Thus, on a continuum with votaries of humanistic art of teaching at one end and that of rigorous science at the other, scientific basis of the art of teaching belongs near the middle of the range.

Following are a few more definitions of teaching which would be helpful to analyze the nature of teaching :

Henderson (1963, p. 1007) : Teaching can be conceived as the ternary relation : x teaches y to z. Expressed in the notation of relation theory, this becomes (x, y) T z or more generally T (x, y, z). As popularly conceived, the domain or 'x' is the set of persons who act as teachers; the domain of 'y' is the set of knowledge, beliefs or skills selected by a teacher and the domain of 'z' is a set of individuals—human and other animals capable of modifying their behaviour as a result of experience—who are taught by a teacher.

Gage (1963a, p. 96) : By teaching we mean...any interpersonal influence aimed at changing the ways in which other person can or will behave. The influence has to impinge on the other person through his perceptual and cognitive processes.

Hough and Duncon (1970, p. 2) : Teaching is an activity—a unique professional, rational and humane activity in which one creatively and imaginatively uses himself and his knowledge to promote the learning and welfare of others.

Smith (1971, p. 20) : Teaching consists of a succession of acts by an individual whose purpose is either to show other persons how to do something or inform them that something is the case.

Joyce and Weil (1972, p. 3) : We think of teaching as a process by which teacher and students create a shared environment including sets of values and beliefs (agreement about what is important) which in turn color their views of reality.

Cooney *et al.* (1975, p. 10) : When one studies what teachers do, one finds that some activities are related closely to teaching; demonstrating, explaining, defining, correcting students' mistakes, providing motivation, and assessing students' progress are some of these.

The main streams of thought that flow through these definitions are the following :

1. Teaching is an ordered set of teacher behaviours.
2. Ordering of this set is purposeful.
3. Teaching aims at bringing about change in the learner.

4. The change occurs through perceptual and cognitive processes of the learner.
5. The change manifests itself chiefly in the form of cognitive reorganization.

1.1.3. Central Objective of Mathematics Teaching

Having conceived mathematics as an embodiment of closely related concepts, attainable through the process of pure thought; and teaching as purposeful behaviour of the teacher aimed at bringing about change in cognitive reorganization of the learner, the meaning of mathematics instruction that emerges is—purposeful behaviour of the teacher to induce thinking in order that mathematical concepts are assimilated in cognitive structure of the learners. The central objective of mathematics teaching is thus, "Acquisition of Mathematical Concepts."

This assertion has been corroborated by Fehr (1966, p. 235) when he said, "The acquisition of basic concepts is a necessary and desired outcome of mathematical instruction." Weil and Joyce (1978, p. 29) have also remarked: "Helping children learn concepts and teaching them how to learn concepts is a fundamental purpose of schooling."

Concepts are the vehicles of thought. Therefore an individual can think mathematically only if he has assimilated mathematical concepts into his cognitive structure. It was for this reason that 'Acquisition of Concepts in Mathematics' was singled out as outcome variable in the present study.

1.2.0. Theory, Model, Strategy and Method

As has been indicated earlier in Section 1.0.0., there has been a continuous impact of psychological theories on the methods of teaching in general and teaching mathematics in particular. A theory becomes amenable to practical classroom utility only when it is transformed into a method of teaching. Model and strategy could be considered as intermediate steps in the process of such transformation.

1.2.1. Theories of Teaching

Literature on educational psychology abounds in theories of learning. According to Gage (1964, p. 268):

> Theories of learning will have greater usefulness to education when they are transformed into theories of teaching... While theories of learning deal with the ways in which an organism learns, theories of teaching deal with the ways in which a person influences an organism to learn.

Bruner (1966, p. 40) also struck the same note when he wrote :

> ...theories of learning and of development are descriptive rather than prescriptive. They tell us what happened after the fact.... A theory of instruction, on the other hand... is concerned with how what one wishes to teach can best be learned, with improving rather than describing learning.

Smith (1960) presented a rationale for formulating theories of teaching that are wholly independent of the theories of learning. Ausubel (1968, p. 14), on the other hand, stressed that "theories of learning and theories of teaching are interdependent rather than mutually exclusive."

Many theories of teaching have been suggested by psychologists such as : Henderson (1965), Maccia (1965), Bruner (1963), Strasser, (1967), Scheffler (1965), Walton (1966) and Scudder (1968). These theories, however, have not been as useful to education as they were expected to be, for the reason that they were stated at most abstract level. In respect of Bruner's theory, for example, Rohwer (1970, p. 1396) writes :

> The theory is stated at such a high level of generality that the implementation of its implications in educational practice is left almost entirely to the ingenuity of the schoolman, whatever his capacity. Presently, it can make contact with educational practice only in the hands of brilliant and inventive practitioners.

1.2.2. **Models of Teaching**

Joyce and Weil (1972) have done a great service to educational practice by developing the models of teaching from number of learning theories. Since these models of teaching were stated more explicitly and directly useful for classroom purpose, they attracted greater attention of educational practitioners than the forementioned theories of teaching.

The term 'model' seems to be one of the most ambiguously used ones in the literature on education. Brodbeck (1963, p. 88) lamented that "Model is now frequently used for those theories which are either highly speculative or quantified, or, most likely, both" and Hyman (1971, p. 131) pointed out that, "these two words, theory and model, often are used interchangeably." In the present study, however, the model is not a synonym for theory but a translation of the theory into practical teaching form.

According to Weil and Joyce (1978, p. 2) :

> A model of teaching consists of guidelines for designing educational activities and environments. It specifies ways of teaching and learning that are intended to achieve certain kinds of goals. A model includes a rationale, a theory that justifies it and describes what is good for and why, the rationale may be accompanied by empirical evidence that it works.

Doyle (1979, p. 146) asserted that :

> The common denominator of all the models of teaching is that each attempts to achieve a deliberate instructional goal based on a specific belief about the nature of the child and, most importantly, each suggests a sequence of general participant actions to facilitate learning or produce pupil cooperation in activities that appear to produce learning.

Joyce and Weil (1972, pp. 8-11) group models of teaching into four families :

1. Social Interaction Models;
2. Information Processing Models;
3. Personal Models; and
4. Behaviour Modification Models.

The social interaction models emphasize the relationships of the individuals to society or to other persons. These models of teaching support the efforts of teachers who strive to fulfil the purposes of democratic actions. The models encourage group problem solving, cooperation, inquiry, interpersonal relationships, and de-emphasize competition.

The information processing models are designed to develop inductive mental processes. The focus is on the way students organize information, develop concepts, interpret and propose solutions to problems and use symbols. Some models within this family emphasize the structure of a discipline while others focus on interdisciplinary problems.

The personal models operate on the assumption that as students' self-perceptions are enhanced they are better able to relate to others and become more skilful in information processing.

The behaviour modification models represent systematic plans for sequencing learning tasks and changing behaviour by conscious manipulation of reinforcement.

'Ausubel's Advance Organizer Model', and 'Bruner's Concept Attainment Model'—the foci of the present study—belong to information processing family of models.

Joyce and Weil (1972, pp. 13-16) employ the set of five concepts to describe the models : Orientation provides theoretical background of the model; Syntax includes sequences of events or phases of the model; Principles of Reaction indicate the mode of teachers' responses to students' activities ; Social system signifies the norms with regard to student-teacher roles and relationships; Support system refers to additional requirements beyond the usual human skills, capacities and technical facilities to implement the model.

Following is the brief description of Ausubel's Advance Organizer Model and Bruner's Concept Attainment Model adapted from Joyce and Weil (1972).

1.2.2.1. Concept Attainment Model

This model has been developed from 'A Study of Thinking', the work of Bruner, Goodnow and Austin.

Orientation

In order to cope with tremendously diverse environment human beings engage in the process of categorizing, which means rendering discriminately different things equivalent and responding to them in terms of their class relationship rather than uniqueness. Concept attainment is essential for categorizing and occurs by making decisions about attributes belonging to the categories. An attribute is a characteristic of an object or event. Examples as well as non-examples of the concept can be analysed in terms of attributes.

According to Bruner *et al.* (1956, p. 22) :

> Concept attainment refers to the process of finding predictive defining attributes that distinguishes examples from non-examples. This is different from concept formation—the process in which new categories are formed. Concept formation is essentially the first step *en route* to attainment.

Acquisition of concepts in a school subject involves the process of concept attainment by the students with the help of concept attainment games. Weil and Joyce (1978, p. 32) state :

> The requiremeet for concept attainment games are relatively simple: (the games are) an array of instances or examples that are alike in some ways and different in others. A student encounters these examples and must find out or be told by the teacher whether each instance does or does not exemplify the concept. At each encounter with an instance the student formulates and reformulates a hypothesis about the concept. Each instance or example provides potential information about the characteristics and attribute values of the concept.

Bruner *et al.* (1956, pp. 41-42) identify three types of concepts: conjunctive, disjunctive and relational. Conjunctive concepts are defined by joint presence of several attributes. Disjunctive concepts require the presence of some attributes and the absence of others. Relational concepts, like conjuctive concepts, have several attributes, but these bear some kind of relationship to one another.

Bruner and his associates contend that there are three modes of concept attainment: enactive, ikonic, and symbolic. Enactive mode refers to thinking in terms of actions. Ikonic mode refers to thinking in terms of images. Symbolic mode refers to thinking in terms of symbols. Bruner believes that fairly sophisticated mathematical thinking cannot take place until a child acquires symbolic mode of concept attainment.

Bruner and his co-workers identified six strategies—four selection strategies and two reception strategies—by which subjects attain concepts. The selection strategies are : (1) simultaneous scanning; (2) successive scanning; (3) conservative focussing; and (4) focus gambling.

In simultaneous scanning strategy, the subject uses each instance encountered as an occasion for deducing which hypotheses are tenable and which have been eliminated. In successive scanning strategy, the subject formulates a single hypothesis and tests it against the incoming instances. In conservative focussing strategy, the subject begins with a positive instance and then successively searches for other instances, one at a time, that are similar in all respects but one. In focus gambling strategy, the subject begins with a positive instance as a focus and then successively looks for instances that change more than one attribute at a time.

The reception strategies are : (1) wholist and (2) partist. In wholist strategy, the subject takes the first positive instance and uses it as a guide, comparing all the attributes of the first instance to those of subsequent instances and modifies the hypothesis accordingly. In partist strategy, the choice of the hypothesis is based on only part of the initial example. If the initial hypothesis is not confirmed, the partist refers back to all previous instances and changes the hypothesis.

Joyce and Weil (1972, p. 112) contend :

> We have developed a concept attainment teaching model to serve three purposes: One is to teach students about the nature of concept. A second is to teach students to be more effective in attaining concepts (finding out categories which others use to organize the environment) and forming concepts (developing new categories). The third purpose is to teach specific concepts.

Syntax

Phase One : Presentation of Data and Identification of Concept.

This phase involves presenting data to the learner. Each unit of data is a separate example or non-example of the concept. The learners are informed that there is one idea that all the positive examples have in common; their task is to develop a hypothesis about the concept. The instances are arranged in a pre-arranged order and are lebelled "yes" or "no". Learners are asked to compare and justify the attributes of the different examples. Finally, they are to name the concept and state the rule or definition of the concept according to its essential attributes.

Phase Two : Testing Concept Attainment.

In this phase, the students test their attainment of the concept, first by correctly identifying additional unlabelled examples of the concept and then by generating their own examples.

Phase Three : Analysis of Thinking.

In this phase, students begin to analyse the strategies by which they attain concepts.

Principles of Reaction

During the flow of the lesson, the teacher is supportive of the students' hypothesis, emphasizing, however, that they are hypothetical in nature and creating a dialogue in which the major content is a balancing of one person's hypotheses against another's. In the later phases of the model, the teacher encourages analysis of the merits of various strategies rather than attempt to seek the one best strategy for all students in all situations.

Social System

The teacher selects the concepts, selects and organizes the material into positive and negative examples, presents data and designates it as belonging or not belonging to the concept. In the latter stages, the teacher shifts to an analytic role, but again draws the students into analysis being exceedingly careful not to provide them with the criteria by which they can judge their strategies.

Support System

Concept Attainment lessons require material designed *in* such a way that concepts are embedded in the material. The material must be organised so that positive and negative examples are pointed out to the students.

1.2.2.2. Advance Organiser Model

This model has been developed from the theory of meaningful reception learning propounded by Ausubel.

Orientation

Ausubel espouses the view that each of the academic disciplines has a structure of concepts which froms the information processing system of the discipline. He conceptualizes the discipline as levels of hierarchically organized concepts that begin with perceptual data at the bottom and proceed through increasing levels of abstraction until the most abstract concepts appear at the top.

Joyce and Weil (1972, pp. 166-167) state :

> The task of the school, according to Ausubel, is to identify clear, stable, and organized bodies of knowledge within the discipline. The most important kind of learning which the school can foster is the acquisition of these bodies ...As the learner acquires clear, stable and organised bodies of knowledge, these constitute in their own right the most significant independent variables influencing the meaningful learning and retention of new subject matter.

Ausubel distinguishes between meaningful learning and rote learning. Meaningful learning implies that what has been learned is intellectually linked and understood in the context of what was previously known. It also implies that this knowledge can be transformed and applied creatively in new and rather novel situations. Rote learning, in contrast, typically lacks conceptual and critical approaches to the acquisition of information. It usually does not prepare the learner to transform or apply the knowledge in new context. It is also highly subject to forgetting.

According to Weil and Joyce (1978, p. 202), "The advance organizer model is designed to strengthen students' cognitive structure. By cognitive structure, Ausubel means persons' knowledge of a particular subject matter at any given time and how well organized, clear and stable it is." Two principles are suggested for programming content in the subject fields in such a way that the concepts become a stable part of students' cognitive structure. The first is progressive differentiation and the second is integrative reconciliation.

Progressive differentiation means that the most general ideas are presented first, and these are progressively differentiated in terms of detail and specificity. Integrative reconciliation simply means that new ideas should be consciously reconciled and integrated with previously learned content.

In the actual teaching situation, the primary way of strengthening cognitive structure and enhancing retention of new information is through the use of 'Advance Organizer'. According to Anderson (1967, p. 158) :

> Ausubel believes that learning from meaningful verbal material can be facilitated by providing the student with advance organizers which help the student to integrate the new material into his existing cognitive structure. The organizers compare and contrast the new material with what the student already knows.

Ausubel (1963, p. 29) describes advance organizer in the following way. "Advance organizers consist of introductory material at a higher level of abstraction, generality and inclusiveness than the learning task itself." According to Ausubel, Novak and Hanesian (1978, p. 148), "The principal function of the organizer is to bridge the gap between what the learner already knows and what he needs to know so that he can learn the task at hand". Advance organizers are generally based upon the major concepts, propositions, generalization, principles and laws of the discipline.

Two types of organizers have been recommended. Expository organizer provides a general model of class relationships as a general subsumer for a new class, subclass and species before more limited subsumers are provided for the particular subclass of species. Comparative organizers, on the other hand, are used most with relatively familiar material. They are designed to integrate new concepts with basically similar concepts existing in the cognitive structure.

According to Ausubel (1968, pp. 511-516) :

Older (school-age) children, as well as adolescents and adults, acquire new concepts through the process of concept assimilation. That is, they learn new conceptual meanings by being presented with the critical attributes of concepts and by relating these attributes to relevant established ideas in their cognitive structure.

He further argues :

> It should be borne in mind that principles of concept formation, based on laboratory studies, are not necessarily coextensive with or even analogus to principles of concept assimilation in mastering subject matter matterial...In learning a new concept as much or more depends on existing properties of cognitive structure, on the developmental status of the learner, and on his intellectual ability as on the nature of the concept itself and the way in which it is presented.'

Syntax

Phase One : Presentation of the Advance Organizer

This phase consists of three activities : clarifying the aims of the lesson, presenting the organizer, and prompting awareness of relevant knowledge and experience in learners' background. While presenting the organizer, the teacher identifies its defining attributes, gives examples, provides multicontext and repeats the terminology of the subsumer.

Phase Two : Presentation of the learning task or material.

In this phase, the teacher makes logical order of learning material explicit to the students. He maintains attention and makes the organization explicit.

Phase Three ; Strengthening of Cognitive Organization.

In this phase, the teacher strengthens the anchoring of the new material in the students' cognitive structure. He facilitates integrative reconciliation of the new material with material in the existing cognitive structure and promotes active reception learning. Finally students' difficulties, if any, are clarified by giving additional new information, rephrasing previously given information, or applying the ideas to a new problem or example.

Principles of Reaction

The teacher's responses to the learners' reaction are guided by the purposes of clarifying the meaning of the new learning material, differentiating it from and reconciling it with existing knowledge, making it personally relevant to the student, and helping to promote a critical approach to knowledge.

Social System

The teacher retains controls of the intellectual structure, as it is necessary to continually relate the learning material to the organizers and to help students discern new material, differentiating it from previously learned material. During the presentation of the organizer and, to a lesser extent, the presentation of the learning material, the interaction is primarily one way, teacher to student. In phase three, however, the learning situation is ideally much more interactive, with students initiating many questions and comments.

Support System

Well organized material is the critical support requirement of this model. The effectiveness of the advance organiser depends upon an integral and appropriate relationship between the conceptual organizer and the content.

1.2.3.

Strategies of Teaching

The term strategy has been referred to by different authors in different ways. Hough and Duncon (1970, p. 164) define a strategy "as a pattern of substantive, managerial or silent behaviour used to facilitate students' attainment of an objective". According to Fraenkel (1973, p. 18), "Teaching strategies refer to operations a teacher performs, in order to involve students in activities to help them learn...". From these definitions, it appears that the term 'strategy' has been used to mean teacher activities to facilitate students' learning.

Smith *et al.* (1967, p. 16) considered two dimensions of a strategy: the treatment dimension and the control dimension. The treatment dimension is concerned with the type and sequence of operations that the teacher and the pupils jointly enter into in setting forth and structuring information in such a way as to disclose the content that is to be learned. The control dimension deals with the operations the teacher uses to guide and control the participation of pupils in performing these operations on the content. If Smith's partition is applied to the models of teaching, the syntax of the model would represent treatment dimension and principles of reaction, social system and support system would roughly determine control dimension. The term strategy, in this study has, however, not been used as a synonym to model but rather as a modified and operational form of the model to suit the specific purpose.

Taba (1969) observed that, "It is useless to study teaching in general as a global process serving all objectives of education. Rather, it seems necessary to identify the particular teaching strategy required by particular type of learning goals."

Weil and Joyce (1978, p. 9) indicated that "Teachers can create many variations on each model, or they can take the essence of a model dropping the syntax and the phases..."

Models of teaching discussed in the previous section represent general format for teaching of concepts in all subjects. However, mathematics being a discipline with its own peculiarity, it was thought necessary to modify the syntax of these models to suit the teaching of mathematical concepts involved in this study. These modified forms or variations of the model were termed strategies.

The treatment dimension of *Bruner Strategy* in this study had the following phases :

1. Presentation of positive and negative examples;
2. Identification of concept by the students;
3. Elicitation of definition from the students; and
4. Analysis of concept attainment strategies.

The treatment dimension of *Ausubel Strategy* in this study had the following phases :

1. Presentation of advance organizer;
2. Presentation of concept by definition;

3. Illustration of concept by examples; and
4. Relation of concept to advance organizer.

Control dimension, that is, principles of reaction, social system and support system of these two strategies were the same as those of their parent models.

In addition to these two strategies, one traditional strategy, forming a control treatment, was employed in this study. It had the following phases :

1. Presentation of definition;
2. Illustration of definition by examples;
3. Memorization of definition by repetition; and
4. Mechanical application of definition to solve problems.

1.2.4. **Methods of Teaching**

According to Broudy and Palmer (1965, p. 9), "Method refers to a set of procedures that are carried out according to some rule." Smith (1971, p. 21) pointed out : "By method is generally meant a particular order imposed upon teaching activity. It is a construction of how teaching ought to be done." Henderson (1963, p. 1007) holds, "The pattern, that is, set of common properties, that a set of behaviour sequence manifest, will be called a method."

The gist of these three definitions is that method involves a sequence of behaviour determined by some rule. This rule must have some theoretical orientation.

Thus, a behaviour sequence formulated on the basis of discovery orientation would be a discovery method and that developed on the basis of expository orientation would be an expository method. Other examples of methods are inductive, deductive, analytic, synthetic, laboratory, lecture, etc.

In teaching-learning situation, however, one single method is seldom used. To achieve an objective of the lesson the teacher has to employ the force of several methods. This purposeful combination of methods constitutes a strategy.

Gerhard (1971, p. 18) went to the extent of observing that "Teaching strategies have been classified as a series of methods ranging from lecture, discussion and recitation to the multi-method, the project, the self discovery and self selection approach." The present investigator, however, thinks that, in order to constitute a strategy, the methods involved should be such that they can be articulated with each other. A strategy may also include some special technique to accentuate the effect of these methods.

Ausubel strategy in this study included Advance Organizer technique in addition to deductive expository method; and Bruner strategy in this study involved the analysis of concept attainment strategies in addition to inductive discovery method.

1.3.0. Cognitive Entry Behaviours

While evaluating the effectiveness of instruction, it is imperative that cognitive entry behaviours of the learners be taken into account. According to Bloom (1976, p. 11) cognitive entry behaviours refer to "the prerequisite learning held to be necessary for task(s) on which instruction is to be provided". Students' previous knowledge of the learning task would, thus, determine their cognitive entry behaviours. But Bloom (1976, p. 52) recognizes general intelligence also as universal cognitive entry behaviour and evinces that "intelligence measures account for about twenty-five per cent of the variance while achievement measures account for fifty to sixty-four per cent of the variance on subsequent achievement measures". On the basis of this, Bloom (1976, p. 109) further asserts :..."the lack of the necessary prerequisite cognitive entry behaviours for a particular learning task should make it impossible for the students to master the learning task requirement, no matter how good the quality of instruction for that task."

Ausubel (1968, p. vi) stresses the importance of previous knowledge on the part of students in the following words :

> If I had to reduce all of educational psychology to one principle I would say this : The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly.

Shulman (1970, p. 64) remarked :

> The students' knowledge of the area in which instruction must proceed will be an important factor....This would

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suggest that certain minimal prerequisite of understanding may be necessary before effective instruction leading to cognitive reorganization can be successful.

Wiviott's (1970) study on geometric concepts provided evidence that high achieving children excel in concept learning. Achievement, on the other hand, has been found to be highly correlated to intelligence. Reid (1964) obtained a correlation ($r=0.85$) between intelligence and achievement in arithmetic for grade six Jamaican school children. Isaacs (1974) also found significant correlation ($r=0.63$) between mathematics achievement and verbal intelligence of third year pupils in Jamaican secondary schools. Sanand-Raj and Krishnan (1980) found the correlation ($r=.631$) between intelligence and achievement.

There have been studies presenting evidence of significant contribution of intelligence in the acquisition of mathematical concepts. Bruce (1976) has shown that children with higher intelligence excel in mathematical concept formation. Sharma and Aggarwal (1980) found that the performance of students on different taxonomic categories of cognitive tasks in algebra varied directly with intelligence. Joshi and Gakhar (1980) also concluded that the variable of intelligence contributes to significant differences in the acquisition of Algebraic concepts.

Previous knowledge and intelligence were, therefore, treated as concomitant variables in the present study.

1.3.1. Cognitive Style

Individuals differ in the ways in which they process information and these differences have repercussions for cognitive functioning. Teaching by particular information processing model, therefore, may have differential effects on the individuals of different cognitive styles. A dimension of cognitive style was included as moderator variable in the present study, in order to observe these differential effects.

According to Goldstein and Blackman (1978, p. 2) :

Cognitive style is a hypothetical construct that has been developed to explain the process of mediation between stimuli and responses. The term cognitive style refers to the characteristic ways in which individuals organize the environment.

Messick (1976) defined cognitive style in terms of consistent patterns of "Organising and processing information". Coop and Sigel (1971) used the term cognitive style "to denote consistencies in individual mode of functioning in a variety of behavioural situations".

There have been numerous approaches to the study of cognitive style under various names such as : Cognitive control (Gardner *et al.*, 1959), field-independence (Witkin *et al.*, 1954), Cognitive complexity (Kelly, 1955; Bieri *et al.*, 1966; Harvey *et. al.*, 1961), Reflection impulsivity (Kagan *et al.*, 1964), Styles of categorization (Pettigrew, 1958) and conceptual style preference (Kagan *et al.*, 1963).

The approach chosen for the present study was conceptual style preference. According to Kagan (1966), "Conceptual style preference is a dimension of cognitive style which refers to the types of conceptual relationships among objects typically formed by the child." Kagan, Moss, and Sigel (1963) defined conceptual style preferences as : "...stable individual preferences in mode of perceptual organization and conceptual categorization of the external environment". These authors identified three types of conceptual style preferences : categorical, descriptive and relational.

Categorical responses refer to the use of common class membership in relating stimuli. This style has been referred to as inferential mode of conceptualization in that the use of abstract label is a means of summarizing the detailed relationship among stimuli.

Descriptive responses may be defined as concepts formed on the basis of shared physical attributes of stimuli. This style has been called an analytic mode of conceptualization in the sense that an individual deals with similarities among the concrete details of stimuli.

Relational responses are those in which functional relationships among stimuli are used in associating stimuli. This style is referred to as a global, contexual, thematic mode in that an individual associates a whole stimulus with another whole stimulus in making an independent functional relationship.

1.4.0. Research on Teaching Methods

According to Tuckman (1972, p. 1), "Research is a systematic attempt to provide answers to questions." Basic research attempts to develop the model or a theory that identifies all the relevant variables in a particular environment and hypothesizes about their relationships. It does not provide immediately usable information for altering the environment. A test or try-out that includes systematic evaluation of the model or a theory comes under the province of applied or action research and it provides immediately usable information for educational practice.

Gage (1963, p. 97) defined research on teaching as, "research in which at least one variable consists of a behaviour or characteristic of teachers". In a preface to the *Handbook of Research on Teaching*, he designated 'a behaviour or characteristic of teacher' as central variable in the research on teaching and classified it into three categories : (1) teaching methods; (2) instruments and media of teaching; and (3) teachers' personality and characteristics.

The research on teaching methods is that in which an investigator studies the relation between the methods employed and expected student behaviour under various hypotheses.

In the light of Henderson's (1963, p. 1007) definition of teaching as ternary relation : T (x, y, z), this kind of research focuses on binary relation x R z abstracted from T (x, y, z). It tries to identify members (x, z) of the relation which are invariant with respect to y. Henderson (1963, p. 1013) recognized three kinds of research of this type.

The first kind of research tests the existence hypothesis in which an experimenter seeks to exhibit the conjunction of an identifiable method and certain behaviours of a student which may reasonably be considered related to the method employed. In other words, an experimenter shows that the method is a sufficient condition for the manifestation of certain behaviour by the student.

The second kind of research tests the uniqueness hypothesis. A researcher shows that a pattern of behaviour by students is associated with a certain method the teacher employs. In other words, the researcher establishes that the method is a necessary condition for the student behaviour in question.

The third kind of research is an attempt to show that the method is both necessary and sufficient condition for certain behaviour on the part of students to occur.

Under this classification the research involved in the present study is the first kind of research.

The research on the method of teaching can also be classified on the basis of number of methods used as independent variables in the study. 'Only one method' research is an argument in favour of one particular method. It advocates efficacy of the method for some specific purpose to achieve. 'Two or more methods' research seeks to compare two or more methods with regard to their effectiveness on some predetermined criterion. Research involved in the present study belongs to the latter category.

Mitzel (1960, pp. 1483-1484) and Dunkin and Biddle (1974, p. 39) have classified variables involved in the research on teaching as follows :

Presage variables : These refer to certain teacher characteristics such as age, sex, social class background and training experience.

Context variables : These refer to the features of the context within which teaching and learning takes place such as grade level, subject matter, size of class and type of community.

Process variables : These variables deal with the ways in which the teachers and pupils behave and interact—think, talk, write, move and relate to one another.

Product variables : These denote the amount of learning or achievement of educational objective.

Six kinds of possible relationships can be conceived of among these four types of variables : Context-process, context-product, presage-process, presage-product, context-presage and process-product.

The research on teaching can be classified according to the kinds of relationships investigated in the study. The relationship investigated in a particular study obviously depends upon the purpose of the study. Since the purpose of the present study was not to analyse the process of teaching but rather to assess and compare the effectiveness of the two strategies as a whole for various criteria of concept acquisition, its major emphasis was on presage-product relationship.

1.5.0. Statement of the Problem

On the theoretical background of foregoing sections, the problem for the present investigation was stated precisely as below :

"EVALUATING DIFFERENTIAL EFFECTIVENESS
OF AUSUBEL AND BRUNER STRATEGIES FOR
ACQUISITION OF CONCEPTS IN MATHEMATICS"

1.6.0. Significance of the Study

Shulman (1970, p. 59) appraised Worthen (1968) study on expository-discovery presentation by saying that "...his investigation possesses a number of characteristics which would well serve as a model for future research in this area" but criticized the work by indicating that neither his discovery treatment followed an important characteristic of Bruner's model, nor his "expository treatment in any way reflected an attempt to apply systematically the tenets of either Gagne's or Ausubel's models".

In the present study all important characteristics of Worthen's research have been maintained. Moreover, Ausubel's model has been employed to represent expository treatment and Bruner's model has been employed to represent discovery treatment. It is, therefore, expected that this study would not only indicate relative effectiveness of the two models but also throw a new light on long-standing expository-discovery controversy.

Ausubel and Bruner strategies, evolved from the respective models by modifying their syntaxes to suit the teaching of mathematical concepts, were compared with traditional strategy in the form of control treatment. This would help to determine the efficacy of these models in Indian classroom conditions.

The study focussed on the central objective of mathematical instruction, *viz.*, the acquisition of mathematical concepts. Multiple criteria of knowledge, transfer, heuristic transfer, short-term retention and long-term retention were employed to test the attainment of the objective.

Intelligence and previous knowledge which account for substantial variance on criterion measures of concept acquisition were statistically controlled in this study.

Since cognitive style of the student could affect his information processing mode, it was considered necessary to ascertain effectiveness of the strategies separately for the students of different conceptual style preferences. This was expected to provide some indication regarding probable matching of conceptual style preference with the two instructional strategies.

The experiment was conducted in actual classroom situations in order to increase the external validity of whatever evidence that might accrue from the study.

Teacher fidelity to prescribed treatments was measured by specially designed 'Observer's Rating Scale' to obviate the usual criticism of non-adherence to treatments in comparative studies of this type.

The rationale for the study has been given in detail at 2.9.0. supra.

1.7.0. Objectives of the Study

The objectives of the present study were spelled out as below :

Principal Objectives

(*i*) To ascertain comparative effectiveness of Ausubel strategy with traditional strategy on the various criteria of concept acquisition in mathematics.

(*ii*) To ascertain comparative effectiveness of Bruner strategy with traditional strategy on the various criteria of concept acquisition in mathematics.

(*iii*) To ascertain relative effectiveness of Ausubel and Bruner strategies on the various criteria of concept acquisition in mathematics.

Subsidiary Objectives

(*i*) To ascertain effectiveness of Ausubel strategy in the acquisition of concepts in mathematics, separately for the students of different conceptual style preferences.

(*ii*) To ascertain effectiveness of Bruner strategy in the acquisition of concepts in mathematics, separately for the students of different conceptual style preferences.

(*iii*) To ascertain relative effectiveness of Ausubel and Bruner strategies in the acquisition of concepts in mathematics, separately for the students of different conceptual style preferences.

1.8.0. Assumptions Underlying the Study

1. Ausubel's Advance Organizer Model and Bruner's Concept Attainment Model stand as the choicest representatives of Expository and Discovery methods respectively.

2. Syntaxes of these models can be suitably modified for the developmental level of the students and the nature of concepts to be taught.

3. The teacher can be programmed to a great extent to teach according to these models.

4. Mathematical concepts of set, relation, function, limit, continuity and derivative can be taught by both of these models.

5. Relative effectiveness of these two models can be judged on the basis of product objectives.

6. Concept acquisition in mathematics can be assessed by the criteria of knowledge, transfer, heuristic transfer, short-term retention and long-term retention.

7. Teacher-made tests are more useful than standardised tests for the measurement of outcome set forth in a given study.

8. There exist intra-teacher variations which have random effect on the acquisition of concepts by the students.

9. Teacher behaviours are only partially controlled by students' responses and to a large extent by the method adopted.

10. Non-verbal cues (gestures, movements, etc.) have random effect on concept acquisition by the students.

11. Teacher fidelity to prescribed treatments can be measured on the basis of observation and rating by trained observers.

12. Cognitive structure variables affect the subsequent learning of concepts.

13. Information processing mode of the students it determined by their cognitive style.

14. Intelligence and previous knowledge can be controlled statistically.

15. Error variance due to random fluctuations are self-compensating.

1.5.0. Scope and Delimitations of the Study

1. The study was confined to eleventh grade students enrolled in three higher secondary schools of the Nagpur city during 1981-82.

2. Most of the students were from the lower middle class of the society.

3. Only male students were involved in the study.

4. The number of students in Ausubel, Bruner and control treatment groups were limited to thirty-seven, thirty-six and thirty-eight respectively.

5. The study was limited only to some concepts selected from the syllabus of eleventh grade mathematics.

6. Total teaching time for each treatment was twenty-five class periods.

7. Advance organizers in Ausubel strategy were presented orally. Written organizers were not used.

8. Bruner strategy in this study was based on reception oriented concept attainment model. Selection oriented or unorganised material models were outside the purview of this study.

9. Only the instructional effects of the models were tested in this study. Nurturant effects were not observed.

10. The study centred only on the product objectives of instruction. Process variables were not investigated.

1.10.0. Definitions of the Terms Used

Evaluation : Qualitative assessment by means of statistical significance.

Differential Effectiveness : Relative effectiveness for different student characteristics (Cognitive Style).

Ausubel Strategy : Ausubel's Advance Organizer Model with modified syntax as given in section 1.2.3.

Bruner Strategy : Bruner's Concept Attainment Model with modified syntax as given in section 1.2.3.

Traditional Strategy : Control treatment in the form of traditional mode of teaching as given in section 1.2.3.

Concept : A form of mental construct (taken as undefined).

Mathematical Concept : Terms indicating generalizations about systematic patterns of mathematical relations.

Acquisition of Concepts : 'Concept Attainment' or 'Concept Assimilation' (as the case may be) determined by knowledge, transfer, heuristic transfer, short-term retention and long-term retention of the concepts.

Concept Knowledge : Recognition of concepts learnt.

Concept Transfer : Application and extrapolation of concepts learnt in situations similar to those presented during teaching.

Heuristic Transfer : Application, analysis and synthesis of the concepts learnt to discover new relationships in situations different from those presented during teaching.

Short-Term Retention : Recall of concepts learnt after two weeks' period.

Long-Term Retention : Recall of concepts learnt after eight weeks' period.

Previous Knowledge : Knowledge of related concepts acquired in previous two grades and expected to have been retained.

Intelligence : An ability of observation and clear thinking, operationally accepted as measures on Raven's Standard Progressive Matrices.

Cognitive Style : Conceptual Style Preference classified as categorical, descriptive or relational.

CHAPTER II

Review of Research

2.0.0. Prologue

Theoretical aspects of the study were presented in the first chapter. This second chapter deals with review of previous research in the areas related to that of the present study.

The specific reasons for reviewing the research would be the following :

(1) It provides a background for several conceptual and functional details;

(2) reveals the gaps in the area and helps to specify the scope of the study;

(3) helps to justify the relevance of the problem;

(4) enables the identification and definition of variables;

(5) aids the formulation of hypothesis;

(6) helps to programme the approach;

(7) helps to select the suitable instruments; and

(8) helps to develop the whole design of the problem at hand.

In short, review of related research offers a better perception of the gaps in knowledge and so provides an opportunity to plan the intended study more precisely and meaningfully.

There appears to be two ways of reviewing the research :

(1) Impressionistic approach; and

(2) Scientific approach.

In an impressionistic approach, the investigator reads through the various findings and reaches a series of impressionistic conclusions. In a scientific approach, the researcher applies precise analytic procedures to scan the studies, finds the gaps in previous research, and designs a study to bridge the gaps. The present researcher was continuously aware of the values of both approaches and has been careful to capitalise on them as per need.

As indicated earlier in section 1.2.4., Ausubel strategy in this study involves deductive expository method preceded by an advance organizer, and Bruner strategy consists of inductive-discovery method using examples and non-examples followed by analysis of concept attainment strategies.

The review, therefore, includes studies on inductive-deductive as well as expository-discovery methods. A few studies involving cognitive style variable have also been given. The past studies on Ausubel's model are mainly those which test the facilitating effect of advance organizer on learning and retention. The studies on Bruner's model, on the other hand, concentrate on finding optimum number of examples-nonexamples or exemplification moves for the attainment of concepts. Only those studies which seemed to have some relevance to the present problem have been reviewed.

A search for the study directly comparing Ausubel and Bruner models in toto did not succeed. A few studies involving some important facets of these two models were, however, available on record and have been reviewed.

2.1.0. Classification of Studies

In order to provide some semblance of organization, the studies have been classified as follows :

(*a*) Expository-Discovery Studies in laboratory conditions;
(*b*) Expository-Discovery Studies in class-room situations;
(*c*) The studies involving cognitive style dimension;
(*d*) The studies involving 'Advance Organizers';
(*e*) The studies involving exemplars-nonexamplars; and
(*f*) Expository Discovery studies involving some elements of Ausubel and Bruner models.

The classification, however, is not mutually exclusive and there may be some amount of overlapping in different categories.

2.2.0. Expository-Discovery Studies in Laboratory Conditions

Gagne and Brown (1961) conducted an experiment on summation of series tasks. Thirty-three male students of ninth and tenth grades were involved in the study. Three treatments—rule-example, discovery, and guided discovery—were employed. The principles were learned by the subjects on one day and learning process was repeated again the next day in order to ensure that successful learning has occurred. Their results showed the best performance for the guided discovery group, the worst performance for the rule-example group and intermediate performance for the discovery group.

Wittrock (1963) studied discovery learning with an experiment on deciphering sentences using four types of groups. The groups were :

(1) Rule given-Answer given (Rg-Ag);

(2) Rule given-Answer not given (Rg-Ang);

(3) Rule not given-Answer given (Rng-Ag); and

(4) Rule not given-Answer not given (Rng-Ang).

It was found that the groups which were given the rule decoded more sentences than the groups which were not given the rule. When the rule was not given, giving the answer enhanced learning. The Rng-Ang showed a higher retention score than learning score while the other groups showed lower retention score than learning scores. Similar results were obtained by Haslerud and Meyers (1958) and Kersh (1958).

Guthrie (1967) used a coding task and example only, example rule, rule example and control group of subjects. On a retention measure he found no differences between the performance of the three experimental groups although each was significantly superior to the control group. On a near transfer task involving the application of the similar types of rules the example-only and example-rule groups were superior to the rule-example and control groups. On a remote transfer task involving the application of different types of rules the example-only group was superior to all other groups.

Hershfeld (1975) examined the influence of different instructional treatments on learning, retention and transfer of numerical series. The treatments included a no-information treatment, a general rule treatment that provided information about rules relating entire classes of series and a specific instant treatment that provided information specific to the relationships within each individual series. The specific instant treatment group recalled more series than either of the other two groups.

2.2.1. An Overview of Laboratory Studies

It is difficult to draw any conclusion about the efficacy of expository and discovery methods for learning, retention and transfer from these studies.

Gagne and Brown (1961) reported discovery treatment to be superior to expository treatment for learning the task whereas Hershfeld (1975), Wittrock (1963), Haslerud and Meyers (1958) and Kersh (1958) showed the superiority of expository treatment over discovery treatment for the purpose.

For retention, Wittrock (1963), Haslerud and Meyers (1958) ana Kersh (1958) found discovery method to be superior but in Guthrie's (1967) study these two methods were judged to be equally effective.

In case of transfer, Guthrie (1967) showed that discovery treatment was more effective than expository for near transfer, whereas for remote transfer giving an example only proved to be better than giving a rule before or after the example.

The evidence, therefore, is inconclusive.

2.3.0. Expository-Discovery Studies in Classroom Situations

Cummins (1960) used discovery approach to teach first quarter calculus to freshmen. He designed two tests for the experiment. Test one was for discovery group and test A was for traditionally taught group. The scores on test one for the discovery group were found to be significantly higher than the scores for the traditional group. No significant difference was found in the scores on test A. The results, thus, showed that discovery trained students would do better on a discovery type test.

In addition to a discovery learning group and a didactically taught control group, Price (1967) included in his study a group of subjects who learned mathematical concepts and principles by discovery and then made use of other materials which were especially designed to aid in transfer of mathematical thinking to problems of the real world. Students in this latter discovery transfer group showed significant gains on the Watson-Glaser test of critical thinking but discovery and control group did not. The investigator concluded that systematic experience beyond discovery seemed needed if gains in critical thinking were to be achieved.

Francis (1975) investigated the effects of learning by discovery and verbal reception learning on forty-eight students from three different grade levels (Grades 1, 3 and 6) for simple and complex concept learning tasks. The results indicated that discovery subjects took significantly longer than verbal reception learning subjects to reach the original learning criterion. On all measures of retention and transfer for both simple and complex tasks and for subjects at each grade level, those learning by verbal reception method were superior to those who learned by discovery. Francis noted that the sample of subjects was taken from schools in which the expository method was the basic method of instruction. The superiority of the verbal reception method might have been due to prior familiarity with that method and the lack of prior discovery experience.

In Cantor's (1982) study, seventy-two Kindergarteners participated in an instructional session involving reception, discovery and control conditions. Two immediate and two three-week delayed criterion tests on simple probability events were administered to all the three groups. Analysis of the results did not show superiority of discovery learning over the reception learning. In the case of both the reception and discovery groups prediction performance on the two criterion tests, identical in type to the instructional tasks, was consistently superior to that on the two criterion tests differing in type from the training task. Significant instructional condition effect occurred predominantly, though not exclusively, in the criterion tests identical in type to the learning task. The study, thus, indicated that both reception and discovery learning were equally effective with regard to performance on the tasks similar to the training task and equally ineffective for the performance on the tasks differing from the training task.

Tate (1978) compared the effectiveness of three instructional strategies : lecture demonstration, lecture laboratory and mediated. The sample ($N=86$) consisted of four intact classes of junior college, three taught by the three experimental strategies and the fourth served as a control group. All classes were taught identical content, the metric system, by the researcher himself. Analysis of variance of the data showed that the three strategies were equally effective for teaching the metric system.

Jolly (1978) studied the effect of laboratory approach on teaching the concepts of perimeter, area and volume to seventh grade students. Four existing classes were selected and treatment assigned to them randomly. Two classes studied the concepts in mathematics laboratory using manipulative aids and investigator developed mimeograph material. Two classes constituted control groups and studied the same concepts in a traditional lecture discussion method. All data for the study were collected by using investigator developed instruments which served as both the pretest and post test. Analysis of covariance indicated that laboratory approach was as effective as lecture discussion method for teaching the concepts to average seventh grade students.

Putt (1979) explored the efficacy of two instructional methods for problem solving. One method followed an instructional model based upon Gagne's information processing view and Polya's stages of problem solving. Heuristic strategies for solving process problems were taught by this method. A second method also gave students experience in solving process problems, without teaching heuristic strategies. The two experimental classes of fifth graders received eight instructional sessions of forty-five minutes' duration over a period of four weeks. Tentative conclusions based on this exploratory study were : (*a*) either method of instruction promotes increased problem solving performance and improved attitude towards problem solving and (*b*) the method of teaching heuristic strategies promotes desirable behaviour related to problem solving.

Gupta (1979) evaluated the effectiveness of analytic-synthetic and traditional (Narration-Explanation) methods for teaching high school geometry. The sample was chosen from intact classes of eighth and ninth grade and full prescribed course was taught by the researcher to both experimental and control classes for the duration of eight months. Bigroup rotational design was used for the study and criterion tests were developed by the researcher himself. Analytic-Synthetic method was found to be superior to Narration-Explanation method for overall achievement by ninth grade students. The two methods were found equally effective for the overall achievement of eighth grade students. The two methods were also found equally effective for the attainment of knowledge, comprehension, application and skill by eighth grade students and for the application objective by ninth grade students.

Kornriech (1969) investigated the acquisition of 'focussing strategy' for solving concept identification problems. In her study three groups of students received different amount of information while solving twenty-four concept identification problems. Discovery group was told nothing, guided discovery group was told every four problems that their approach was incorrect. Programmed instruction group was told one of the rules which defined the strategy every four problems. It was found that significantly more students in the guided discovery group acquired the strategy than in the other two groups. This study suggested that if the training of inquiry skills is systematically built into the teaching programme a difference favouring the discovery method may well occur. Suchman (1964) also arrived at the same conclusion in his comparative study of inquiry training method and traditional teacher-directed method for teaching the principles of physics.

Miyan (1982) studied the effects of guided-discovery, tell and do and pure discovery on the development of creativity of the students. He found that there were no significant differences in favour of any of these methods with respect to mathematical creativity; development of fluency and flexibility, covergent and divergent thinking. But guided discovery group was superior with respect to enhancement of originality than the other two groups. This study involved different types of criterion variables such as piagetian tasks, logical reasoning, creativity and achievement in a particular subject. Therefore it was very difficult for the researcher to synthesize the results. There seemed to be theoretical rationale for selecting the criterion variables for testing the efficacy of expository and discovery methods of teaching mathematics.

Worthen (1968) conducted a study of discovery and expository presentations for teaching mathematical concepts. 432 students of fifth and sixth grade from sixteen classes in eight elementary schools were involved in the study for an instructional period of six weeks. Prior knowledge of students was controlled by imparting pre-treatment instruction in pre-requisite mathematical concepts for a period of two months. Each teacher was thoroughly trained and was required to teach by each method which increased the extent of experimental control. Amount of verbalization in the teacher's presentation was controlled. The degree to which teachers adhered to the prescribed teaching methods in each treatment was assessed by a questionnaire administered to pupils and observer's rating scale rated by trained observers. All criterion tests were designed by the researcher and included both intellectual as well as attitude measures. Results were analysed by using a technique of analysis of covariance taking intelligence, arithmetic computation and arithmetic problem solving as constant covariates in the analysis of each dependent variable. Pretest scores were used as additional covariates in analysis of the post test of each instrument administered in both the pre-and post-test series. Post-test scores on concept knowledge test were used as additional covariates in the analysis of the concept retention and concept transfer tests. The analysis showed that expository learning group was superior to discovery learning group on concept knowledge test administered immediately after instructional period; but on retention tests given after five and eleven weeks discovery group was found to be superior to expository group. There were no significant differences between the scores of the two groups on transfer test and attitude scales. However, discovery group was found to be superior on both oral and written heuristic transfer tests. Reanalysis of Worthen's data by Worthen and Collins (1971), however, suggested equal effectiveness of the two methods on all criterion measures.

Olander and Robertson (1973) studied the effectiveness of discovery and expository methods in fourth grade mathematics. 190 pupils were taught for thirty-one weeks through discovery approach while another 184 used an expository approach. The thirteen teachers were rated on a scale measuring the degree to which each succeeded in using the method assigned. Results showed that subjects taught by expository method were significantly better in computation and subjects taught by discovery method were significantly better in retention of ability to apply mathematics. Subjects scoring lower on the pretest on computation and application improved more under expository approach and those lower on concepts gained more under discovery approach. On principles and relationships the discovery subjects gained at a greater ratio throughout the study.

Loomer (1976) explored the differential effects of small group heuristics and expository learning in calculus. The investigator taught two intact classes one by each method for one semester. In small group heuristic class students worked in groups of three or four to discover the ideas of calculus using instructional material containing questions for investigations. Exploration of concepts preceded formation. The teacher acted as a consultant giving suggestion for solving problems only when asked. In the expository class the teacher acted as primary source of mathematical knowledge. Formation of concepts preceded exploration. Models of teacher behaviours were carefully prescribed for the two methods and teacher fidelity to the prescribed models was measured by an inventory that recorded students' perception of the teacher behaviour with respect to models. Each of the evaluation instruments in the study was examined for its content validity and reliability. Measurements were made both immediately after instruction and one month later. Standard analysis of covariance indicated that after instruction there were no significant differences between the two classes on overall calculus achievement or the calculus achievement at the computation-comprehension or applicability-analysis cognitive levels; on mathematical problem solving achievement; or on attitude toward mathematics or problem solving. One month later the evidence again indicated no significant differences between the two classes on any of the calculus achievement measures but indicated significant difference favouring the expository class on problem solving achievement.

2.3.1. An Overview of Classroom Studies

The studies vary greatly over the grade levels, subject matter, criterion measures, sample size, duration of experiment and the treatments. Grade level ranges from Kindergarten (Canter, 1982) to college freshmen (Cummins, 1960; Loomer, 1976; Tate, 1978). Subject matter varies from simple concept learning tasks (Francis, 1975) to the concepts of calculus (Cummins, 1960; Loomer, 1976). Most of the studies have concentrated on knowledge and retention criterion (Loomer, 1976; Olander and Robertson, 1973; Canter, 1982). The only study which uses multiple criterion of knowledge, transfer, Heuristic transfer, retention and also attitude is that of Worthen (1968). Duration of treatment is longer as compared to those of laboratory experiments. Olander and Robertson (1973) experimented for thirty-one weeks and Gupta (1979) for eight months.

Evidence from these studies regarding effectiveness of discovery and expository approaches for the teaching of mathematics has been equivocal. Cantor (1982), Tate (1978), Jolly (1978), Putt (1979) and Loomer (1976) reported equal effectiveness of discovery and expository approaches on knowledge criterion. Worthen (1968) and Olander and Robertson (1973), however, indicated superiority of expository method for the purpose. For retention, Loomer (1976) showed equal effects of expository and discovery methods; Francis (1975) indicated superiority of expository method whereas Worthen

(1968) and Olander and Robertson (1973) argued in favour of discovery method for the purpose. On the criterion of transfer Worthen (1968) obtained no significant difference between expository and discovery treatments but Francis (1975) indicated superiority of expository method for this purpose.

Heuristic transfer appears to have been studied only by Worthen (1968). He reported superiority of discovery group on this criterion. Cummins (1960) showed that discovery trained students had tendency to do better on discovery type tests. Suchman (1964) and Kornruch (1969) pointed out that the training of inquiry skills in discovery lesson might lead to a difference in favour of discovery learning. Price (1967) indicated that systematic experience beyond mere discovery was necessary for gain in critical thinking, but Mixan (1982) found guided discovery, tell-and-do and discovery methods to be equally effective for the development of mathematical creativity.

Although it is difficult to draw definite conclusion from these studies, the general trend appears to be that of equal effectiveness of the two methods (Tate, 1978; Putt, 1979; Gupta, 1979; Loomer, 1976; Jolly, 1978; and Miyan, 1982) for the teaching of mathematics. There has been greater tendency toward using intact classrooms (Jolly, 1978; Putt, 1979; Gupta, 1979; Tate, 1978; and Worthen, 1968) than forming selected groups. An attempt to give didactive group a fair treatment appeared to have been made in some studies (Putt, 1979; Loomer, 1976; Worthen, 1968). Adherence to prescribed teaching models has also been assessed in some studies (Worthen, 1968; Olander and Robertson, 1973; Loomer, 1976).

2.4.0. The Studies Involving Cognitive Style Dimension

Satterly (1979) studied the covariation of cognitive style, intelligence and achievement. He carried out hierarchical factor analysis of the scores of 430 pupils on a matrix of correlation between three measures of cognitive style (analytic-synthetic, field dependence-field independence, levelling-sharpening) and achievement in mathematics, geography and English. Factors of general ability, field independence and levelling-sharpening were identified. Analysis of reduced correlation matrix suggested that field independence shares a small amount of variance with achievement after control for general ability. The findings provided some support for the independence of cognitive style from general intelligence.

Robbinson and Gray (1974) found that for fifth grade children on mathematical concepts subtest of the Iowa test of basic skills, the inferential style was the most important predictor of performance among the three conceptual style preferences. But on the mathematical problem subtest the analytic style was the most important predictor among the three conceptual style preferences. These researchers found a positive correlation between intelligence and each conceptual style preferences. Even after the effect of verbal and non-verbal intelligence were partialled out, there was significant correlation between the mathematical problem subtest of the Iowa test of basic skills and an analytic conceptual style. However, the correlation between inferential style and school learning for girls was substantially reduced after variance attributed to verbal and non-verbal intelligence was removed from school learning variables. The findings of this study indicated that additional variance was accounted for by cognitive style measure beyond that of verbal and non-verbal intelligence in relation to school learning.

Barbara (1973) designed two studies to investigate the effects of analytic-global and reflectivity-impulsivity cognitive styles on the acquisition of geometric concepts presented through emphasis or no emphasis and discovery and expository lessons. The choice of analytic-global and reflectivity-impulsivity cognitive styles was made because of their use in two operations—discriminating attributes and inferring the concepts—involved in concept attainment at the formal level. It was hypothesized that analytic subjects would be able to discriminate attributes better than global subjects and that the reflective subjects would be able to infer the concepts better than the impulsive subjects. For the first study, analytic global seventh graders were identified by the hidden figure test. Then they studied an introductory lesson and a geometry lesson containing or not containing verbal emphasis during class periods on two consecutive days. A test was given on the second day after completion of the lesson. For the second study, reflective and impulsive seventh graders were identified by the matched familiar figure test. Then they studied an introductory lesson and a geometry lesson presented with discovery or expository mode during class periods on two consecutive days. A test was given on the second day after completion of the lesson. The results of the study I were :

1. Analytic subjects performed better than global subjects;
2. Subjects studying the emphasis lesson performed better than subjects studying the no-emphasis lesson;
3. These results were most evident on questions which assessed the discrimination of attributes; and
4. Emphasis lesson did not benefit global subjects more than analytic subjects.

The results of the study II were :

1. Subjects studying expository lesson performed better than subjects studying discovery lesson;
2. This result was most evident on questions which assessed inference of the concepts; and
3. Expository lesson did not benefit impulsive subjects more than reflective subjects.

Threadgill (1977) investigated the relationship between analytic-global cognitive style and didactic and discovery methods of instruction in mathematical concept attainment. He conjectured that analytic students would learn better from a discovery approach while global students would learn better from a didactic approach. The sample consisted of sixty seventh graders, thirty analytic and thirty global. Hidden figure test was used to identify analytic and global students. The subjects from each group were randomly assigned to either a didactic unit or a discovery unit presented by the researcher. The didactic unit involved deductive method with rules stated followed by examples. The discovery unit involved both deduction and induction. Students were required to identify relevant information and to complete rules. Units were completed in two class sessions. The concept taught was traversibility of graphs. Analysis of covariance, using I.Q. as covariate, revealed that on the mathematical concept attainment task :

1. analytic students performed significantly better than global students;
2. there was not significant difference between the relative effects of the two instructional methods; and
3. there was no significant interaction between analytic global cognitive style and two methods of instruction.

Horak (1978) investigated the effectiveness of inductive-deductive teaching methods and their interaction with cognitive style dimension of field-dependence field independence. The sample of the study was 118 pre-service elementary education teachers enrolled in the section of a mathematics course. The subject matter taught was a two-week unit of selected topics from transformation geometry. Deductive treatment followed a rule example paradigm while the inductive treatment followed an example-rule paradigm. In inductive treatment rules or principles were not stated. The criterion measure was an overall achievement test composed of knowledge, application, analysis and transfer subtests. On testing the main effect hypothesis by one way analysis of variance no significant differences ($p. < .05$) were found between the two treatment means on the overall achievement test and the knowledge and application subtests. Significant differences favouring the inductive group were found between the group means on the analysis and transfer tests. No significant interaction was indicated by linear regression analysis between the treatments and cognitive style when the criterion measures were knowledge, application and analysis subtests. For the criterion measure of overall achievement and transfer, however, the data indicated strong support for the existence of Aptitude Treatment Interaction. Inductive method was found to be better instructional procedure for the transfer of learning.

McLeod, Carpenter, McCornack and Skvarcius (1978) investigated relationship between field dependence-independence and expository vs. discovery learning. Treatments were based on two levels of guidance crossed with two levels of abstraction. The topic was numerical system. The results supported the hypothesis that field independent students would perform best when allowed to work independently whereas field dependent students would perform best when given extra guidance.

McLeod and Adams (1980) designed a further study to search for Aptitude Treatment Interaction (ATI) between the two aptitude variables general reasoning and field independence and treatments that differed in both levels of guidance and the use of an inductive or deductive sequence of instruction. The treatmeat that provided a minimal level of guidance and used an inductive sequence was labelled the discovery treatment; the expository treatment provided maximal guidance with a deductive sequence of instruction. Students from three sections of a mathematics course for prospective elementary teachers participated in the study. Complete data were obtained from forty-seven subjects, twenty-four in the expository group and twenty-three in the discovery group. A twenty-item post test that covered all the concepts in the unit was used to measure retention. Fifteen minutes were allowed for the post-test and seven minutes for the retention test. The data were analysed using multiple regression technique. The ATI with general reasoning occurred as predicted on one of the two dependent variables. The expected ATI with field independence did not occur.

2.4.1. An Overview of Cognitive Style Studies

The studies seem to support the contention that cognitive style variable has some contribution over and above general intelligence in the learning of mathematical concepts. Robinson and Gray (1974) indicated that additional variance was accounted for by cognitive style measure beyond that of intelligence. Satterly (1979) also stated that his study provided some support to the independence of cognitive style from general intelligence.

The studies involving field dependence-field independence dimension of cognitive style produced conflicting evidence. Horak (1978) found significant interaction between inductive-deductive methods of instruction and field dependent-field independent cognitive style on transfer test whereas McLeod and Adams (1980) reported no significant interaction between levels of guidance and field dependent-field independent cognitive style. In an earlier study McLeod *et al.* (1978) showed that field-independent performed best on working independently and field-dependent performed best on providing extra guidance.

The studies on analytic-global dimension of cognitive style seem to produce more consistent results. Barbara (1973) and Threadgill (1977) have reported superiority of analytic style for attainment of mathematical concepts. Robbinson and Gray (1974) have, however, indicated that inferential style was the better predictor of performance on mathematical concepts subtest of Iowa test of basic skills.

2.5.0. The Studies Involving Advance Organizers

Doyle (1981) conducted a study with a primary purpose to determine if an advance organizer could anchor a subsuming concept of function in the cognitive structure and a secondary purpose to determine if an anchored subsumer could facilitate learning, transfer and retention and would differentially benefit students of different abilities. Thirty-six sections of ninth grade remedial mathematics were matched by ability as measured by average performance on the first two mid-terms in the course and then randomly assigned to an advance organizer (AO) or control (C) group. While AO subjects studied the advance organizer, C sections graphed a linear function $y=-3x+1$, $-5\leqslant x\leqslant 5$ for increments of .5 in x. The advance organizer was a programmed concept learning device designed for subsuming concept functions and their graphical representation. AO students significantly outperformed C students overall and at both the formal reasoning levels. Reasoning ability was found to be a significant correlate of performance on advance organizer test. There was no evidence that the organizer differentially benefited students of different abilities. With certain limitations, it was concluded that :

1. An advance organizer can anchor a formal level mathematical concept;
2. Logical reasoning level is a significant correlate of ability to understand the organizer;

3. The anchored subsumer facilitates learning, transfer and retention; and
4. An advance organizer can help in teaching for transfer.

Goodman (1977) studied the effects of Advance Organizer and Generative Processing Cues on the learning of selected mathematical concepts. The four treatments in this study were :

1. Advance Organizer consisting of material more general, abstract and inclusive than the learning material;
2. An example of a problem that can be solved using strategies and instruction to write summaries for each learning section (generative processing cues);
3. A combination of the first two; and
4. A historical passage.

196 ninth and tenth grade geometry students were used as subjects. The content was adapted from contemporary secondary school mathematics. A criterion test constructed by the investigator was used to measure the subjects achievement. The test was divided into two levels. Level I questions were classified as computation-comprehension items and level II questions were classified as application-analysis items. The results of this study showed no significant effect due to treatment and no significant interaction. There was a significant effect due to mathematical ability.

Zakkour (1977) designed a study to examine the manner in which students' personality types affect the extent to which cognitive organizers facilitate the meaningful reception and retention of a new mathematical task. Six intact classes of mathematics for the management, social and life sciences at the University of Oklahoma were randomly assigned into three equal groups; then the treatments were randomly assigned to these groups. A pretest was administered to eliminate from the study those students with previous knowledge of the learning material and to ensure relative equality of groups. The final sample consisted of eighty-eight students. The students in the advance organizer group received a thirty-minute organizer prior to actual presentation of matrices. During the last thirty minutes of the final day of treatment, the post organizer group received the post-organizer. A test for mastery of organizer was attached to both advance and post organizers. The experiment covered eight fifty-minute class sessions. A twenty-five item achievement test was administered during the ninth class session and a twenty five-item retention test was administered three weeks later. Results of 3×2 factorial analysis of variance of the data revealed that :

1. There were no significant differences in achievement and in retention test scores that could be attributed to an interaction between treatments and personality types; and
2. Cognitive organizer did not provide any facilitating effect in the learning and retention of mathematics as was shown by the lack of significant differences among groups and between opposite preferences for each personality dimension.

Smith (1976) studied the effects of organizers and abstract reasoning levels on learning and retention of post-secondary mathematics students. The sample consisted of 235 students enrolled in freshmen level courses at Louisiana State University. Abstract reasoning level of each subject was determined by administering Abstract Reasoning Subtest of Differential Aptitude Test. Subjects were randomly assigned to treatment cells of high abstract reasoning ability group and low abstract reasoning ability group. A 2×2×2 factorial design was used to allow for investigation of interaction. Findings indicated that only students of low abstract reasoning ability were significantly affected by an organizer or combination of organizers. On the learning test a significant interaction of advance organizers and post-organizers was reported and on retention test a significant main effect of advance organizer was reported. It was concluded that above average students did not appear to be significantly affected by advance organizers but below average students were significantly affected by the presence of advance organizers. Advance organizers seemed to have effect on long range retention.

Satterly and Telfer (1979) tested the hypothesis that use of an advance organizer would interact with field independent cognitive style. More specially the hypothesis referred to performance in learning and retention. The results showed that the mean score of pupils who had received advance organizers was not significantly different from the mean score of those who had been tested without learning of advance organizer. Of greatest importance was the significantly higher gain of field dependent pupils under 'advance organizer and its specific reference' treatment over those under no organizer treatment.

In addition to these recent studies there have been quite a sizable number of studies in the past years which claim to have tested Ausubelian Organizer in one experimental setting or the other. Allen (1969), Ausubel (1960, 1978), Ausubel and Youssef (1963), Ausubel, *et al.*, (1968), Cohen (1977), Scandura and Wells (1967) have suggested positive effects of advance organizer on learning and retention. Barren (1971), Bauman, Glass and Harrington (1969), Feller (1973) did not support Ausubel's theory. It should, however, be noted that only one study—that of Scandura and Wells (1967)—among these used mathematical content.

2.5.1. An Overview of Ausubelian Studies

On the basis of studies reviewed, there does not seem to be general agreement on the facilitating effect of Advance Organizer on learning and retention of mathematical concepts. Doyle (1981) showed that Advance Organizer could anchor formal level concept of function in the cognitive structure. In Smith's (1976) study, however, the facilitating effect of Advance Organizer on learning post-secondary mathematics was observed only in case of below-average students. Goodman (1977), Zakkaur (1977) and Satterly and Telfer (1979) concluded that Advance Organizer did not produce facilitating effect on the learning of mathematics.

Doyle (1981) inferred that logical reasoning level was a significant correlate of ability to understand organizer. Goodman (1977) found significant effect of mathematical ability in learning mathematics by advance organizer treatment. Zakkour

(1977) obtained no interaction of advance organizer treatment with personality type but Satterly and Telfer (1979) concluded that field dependent gained significantly higher than field independent on Advance Organizer treatment.

2.6.0. The Studies Involving Exemplars-Non-Exemplars

McMarray (1975) studied the effect of variety and replication of concept instances and pairing of examples on the learning of geometric concepts through the use of four instructional strategies : wide variety-paired (four different rational sets of concept instances, examples and non-examples presented as matched pairs), wide variety single (four different rational sets of concept instances, each instance presented singly), narrow variety-paired (one rational set of concept instances repeated three times, examples and non-examples presented as matched pairs) and narrow variety-single (one rational set of concept instances repeated three times, each instance presented singly). Results indicated that subjects who received wide variety of concept instances performed significantly better on the acquisition test than those who received a narrow repeated variety regardless of whether instances were presented singly or as matched pairs. Presentation of instances as matched pairs resulted in significantly fewer over-generalization errors on the acquisition test than presentation of instances singly for the wide variety condition only. There was no significant difference in over-generalization errors between those subjects who received a narrow variety single instance presentation and those who received a narrow variety matched pair presentation.

Feldman (1975) investigated the effect of two external conditions on learning of principles by children. The external conditions were instruction on the major concepts related in the principle and the number of teaching examples and teaching non-examples of the principle presented with a statement of the principle. Subjects were 159 fifth grade students of average ability and socio-economic status. Two principles from each of two subject matters (mathematics and language arts) served as the content of the learning tasks. Mastery of the principles was determined by tests designed to assess subjects' knowledge of and ability to apply the principles. In general, a rationally selected set of examples and non-examples was found to be more facilitative than one example. Main effects for instruction on concepts related to the principles were not found to be significant.

Charles (1978) designed a study to explore the feasibility of training per-service elementary teachers to use instancing and prompting moves in teaching the mathematical concepts of bilateral and rotational symmetry and to determine the effects of using the instructional moves on mathematical concept acquisition. An instancing move referred to the use of an example or a non-example.

A prompting move referred to a statement by a teacher where the main objective was to draw attention to a relevant or an irrelevant attribute of a concept. Prior to the teaching session, eighteen teachers from mathematics method programme were randomly divided into two groups : trained and control. Each group of nine teachers participated in the training session. Each lesson for each teacher was videotaped, and audiotaped and observed. An analysis of the videotapes, audiotapes and observational data provided scores for the number of instancing and prompting moves used for instruction and a score for the clarity of presentation. Pre-tests and post-tests were administered to the pupils (second grade). Following were the major findings of the study :

1. For teaching at least the mathematical concepts of bilateral and rotational symmetry pre-service elementary teachers can be trained to give more emphasis to non-examples and to prompt the attributes of the concept;
2. The use of non-examples may facilitate the acquisition of certain mathematical concepts; and
3. The number of prompting moves used when teaching a concept may be relatively objective measure of the clarity of teacher's presentation.

Stiff (1978) investigated the relationship between students' pre-instructional knowledge and students' level of concept attainment after exposure to pure characterization and pure exemplification teaching strategies. The content was restricted to one contrived conjunctive algebraic concept 'mat'. Pure characterization strategy of one, four and seven moves and pure exemplification strategy of one, four and seven moves were employed to teach the concept. The amount of information that students know about the contrived concept (relevant knowledge) was manipulated through instruction on subordinate concepts. Students were randomly assigned to one of eighteen treatment booklets. Treatment booklets also contained criterion measures. The experiment involved 326 juniors and seniors from two suburban high schools enrolled in senior level advanced mathematics course and consumed one fifty-minute class period in which students received treatment booklets, read the relevant knowledge instruction, received the instruction on the concept mat and completed the criterion test. The analysis of criterion measures showed that exemplification moves were more beneficial to students with low relevant knowledge and characterization moves were more effective for students with high relevant knowledge. As the number of moves increased from two to four to seven moves no increase in the test performance of subjects was found.

Urbanski (1982) studied the relationship of students' pre-knowledge and students' attainment of a mathematical concept after limited exposure to that concept using a teaching strategy of exemplification moves. The content was restricted to a conjunctive geometric concept—the Hamiltonian line. Sixty-nine juniors and seniors from two high schools enrolled in advanced mathematics course were used as subjects. Students' relevant knowledge was measured by an achievement test given after the presentation of instructional material on subordinate concepts and three groups of low, median and high scores were formed. Students from these three groups were randomly assigned to exemplification strategies of zero, one, seven and thirteen moves. The study used three sequential days of class periods. Post-tests were administered on the third day. In general, it was found that as the amount of relevant knowledge increased so didthe test performance. The number of moves factor containing the zero move, one move, seven moves and thirteen moves affected concept attainment significantly. It was also found that an increase from the seven moves strategy to thirteen moves strategy brings little, if any, increase of test performance on a concept test across all levels of relevant knowledge.

Benton (1977) investigated the relationship between the number of relevant attributes a concept possesses and the number of moves in a teaching strategy necessary to learn the concept. The effects of varying the number and kind of exemplification moves and sequencing in the presentation of concepts was also investigated. A characterization-exemplification-characterization (CEC) strategy was used to teach four conjunctive concepts from mathematical theory of relations for prospective elementary teachers. For each concept the first treatment contained two examples, the second treatment contained one example and one non-example, the third treatment contained four examples and the fourth two examples and two non-examples. In each of these treatments two attribute concepts were presented first. The four remaining treatments were exactly the same except that three attribute concepts were presented first. Results were analysed using non-parametric statistical tests. Gain scores were used throughout the analysis. A significant difference was found between the mean gain scores for two attribute concepts and three attribute concepts. Gain scores for two attribute concepts were significantly higher than those for three attribute concepts. Significant differences were also found in the gain scores for the three levels of cognitive behaviours. Gain scores for analysis, synthesis and evaluation were significantly higher than those for knowledge and comprehension and application. The group receiving four examples and three attribute concepts first achieved significantly higher on three attribute concepts test and also on an application test.

Gage (1977) studied the effects of positive and negative instances on the acquisition of selected algebra concepts as a function of cognitive style. The subjects were randomly assigned to two treatment groups by means of a table of random numbers. Within each group the subjects were classified as high analytic or low analytic on the basis of their performance on the Hidden Figure Test. The subjects in one group were required to learn fifteen concepts in first year algebra by studying positive instances and the subjects in the other group were required to learn the same concepts by studying positive and negative instances. The concept attainment test was developed by the researcher to assess attainment of concepts taught. A two-way analysis of variance was used to test the null hypothesis at the .05 level of significance. The findings of the study indicated that students using positive and negative instances achieved significantly higher than students using positive instances only. Although high analytic subjects scored higher than low analytic subjects, there was no significant difference between the achievement of the two groups. There was no significant interaction between cognitive style and treatment.

In a study conducted over a period of three weeks, Sheel (1981) examined interaction between the field independent/field dependent cognitive style and the use of positive and negative instances on the acquisition of concepts selected from introductory calculus course. The analysis of covariance for the initial achievement data and the retention data revealed no statistically significant differences between the experimental group receiving both positive and negative instances and the control group receiving all positive instances. It was also found that field independent subjects were not superior to the field dependent subjects in initial achievement or retention as hypothesized in the study, nor was there found a statistically significant interaction between the treatments and the cognitive style.

2.6.1. An Overview of Brunerian Studies

Studies range widely on grade level, duration of experiment and content. Grade levels include second grade pupils (Charles, 1978), fifth grade pupils (Feldman, 1975), junior and senior high school students (Stiff, 1978; Urbanski, 1982), first year college students (Gage, 1977) and prospective elementary teachers (Charles, 1978; Benton, 1977). Duration of experiment ranges from one class period (Stiff, 1978) to three weeks (Sheel, 1981). Content taught includes geometric concepts (McMarray, 1975; Charles, 1978; Urbanski, 1982), algebraic concepts (Stiff, 1978; Gage, 1977) and concepts of introductory calculus (Benton, 1977; Sheel, 1981). Conjunctive concepts appear to be a favourable choice (Stiff, 1978; Urbanski, 1982: Benton, 1977).

In general, studies reviewed appear to favour Bruner strategy. Feldman (1975), Charles (1978) and Gage (1977) have shown that presentation of both examples and non-examples was significantly more effective than presenting only examples on the acquisition of mathematical concepts. Sheel (1981), however, found no significant difference between two treatments, one of which was presentation of positive and negative instances and the other a control treatment in which only examples were presented. Benton (1977) found that presentation of four examples and a beginning with higher order concept was more effective than giving two examples and two non-examples. Stiff (1978) and Urbanski (1982) have shown that exemplification could be more effective than characterization if previous training in subordinate concepts is given. Increasing the number of examples beyond a certain limit did not improve concept attainment (Stiff, 1978; Urbanski. 1982). Charles (1978) suggested that attributes of concepts should be prompted, and McMarray (1975) suggested that instances should be presented as matched pairs.

Interaction of positive and negative instances presentation was found to be insignificant with analyticity (Gage, 1977) and with field-independence (Sheel, 1981).

2.7.0. Expository-Discovery Studies Involving Some Elements of Ausubel and Bruner Models

Grabber (1974) directed his study towards gathering further evidence for evaluating the active merits of deductive-expository and inductive-discovery approaches when achievement and retention of major ideas in science were the dependent variables. Deductive treatment presented the major ideas of each thematic area in advance as organizers. The subsuming elements of these ideas were then encountered through assigned chapter readings, guide questions and problems. The terminal activity of each piece of work was the laboratory activity whose main thrust was verification. The initial activity of the inductive programme was the laboratory investigation which provided exposure to attributes of the major ideas. Assigned guide questions and problems and suggested correlative chapter readings followed and served as a basis for codifying and analysing the laboratory data. The culminating activity of each section was a discussion session wherein the teacher sought to guide a discovery (verbalisation) of the major ideas through socratic questioning. Total sample comprised 540 sophomores in twenty-six intact classrooms from four different high schools; each treatment being allotted to thirteen classrooms. The possibility of Hawthorne effect and many other potential sources of variables were controlled. Results were analysed by analysis of variance and covariance taking gain score means as basic statistic. It was concluded that deductive expository and inductive discovery teaching strategies were not significantly different when desired outcome was an understanding of the ideas of science. However, when long-term retention was the criterion, the results indicated superiority of deductive expository method.

Marine (1977) conducted an experimental study to compare example only *vs.* example and non-example strategies and inductive *vs.* deductive strategies of presenting concepts from school geometry. The four strategies were :

1. deductiue-expository, example only;
2. deductive-expository, examples and non-examples;
3. inductive discovery, example only; and
4. inductive-discovery, examples and non-examples.

The sample was chosen from seventh grade pupils. Eight geometry concepts were taught by programmed instruction. Concept identification and concept characterization were the criterion variables. Results were analysed by analysis of variance. No significant differences were found in the concept identification scores attributable to the example only/example and non-example treatment for group of concepts : regular polygon, median, adjacent angles and simple closed curves, but significant ($p < .05$) differences attributable to treatment in concept identification scores were found favouring the example only treatment over the example and non-example treatments for the group of concepts : Congruent polygons, common external tangent, inscribed angle and rhombers. However, the experimenter regarded chance as a still plausible explanation for this difference. On the characterization score significant differences were found favouring deductive strategies over inductive strategies. The experimenter regarded this as very important if generally valid. However, he expressed reservations about the validity of this finding owing to small magnitude of differences and relatively low K. R. 21 reliability (.60) of the criterion measures.

King (1976) designed a study with a purpose to help resolve or explain the apparent conflict between positions expressed by Bruner and Ausubel. The independent variable in this study was instructional sequence and logical reasoning ability was controlled. The three types of instructional sequences were: CA—concrete-abstract, SA—semi concrete-abstract and AA—abstract-abstract. Seventy-four fourth graders were randomly assigned to one of the three instructional sequences. The treatment period was one week. Both immediate and delayed retention were measured by a test at the knowledge and comprehension levels. The test on transfer measured students' ability to solve advanced dependent subconcepts not taught during the experiment. Delayed measures of retention and transfer were administered ten days after the conclusion of the treatment. Three hypotheses were tested utilizing a multivariate analysis of covariance. Each of the null hypotheses relative to differences in the testing groups, treatments and their interaction was accepted.

2.7.1. An Overview of Ausubel Vs. Bruner Studies

The studies reviewed do not strictly adhere to either Ausubel or Bruner model. Grabber (1974) used advance organizer in deductive-expository treatment and also provided exposure to attributes of concepts in inductive discovery treatment but terminating activity in his inductive discovery lesson was not the analysis of concept attainment strategies. In Marine's (1977) study, deductive-expository, example only treatment characterized Ausubel model and inductive-discovery, example-non-example treatment characterized Bruner model but neither 'Advance Organizer' was used in the former nor were the concept attainment strategies analysed in the latter. Although King

(1976) aimed at resolving the conflict between Ausubel and Bruner, neither of his treatments followed the formats of their models. In all the three studies no significant differences were found between the treatments. Grabber (1974) found deductive expository strategy to be superior to inductive-discovery strategy for long-term retention of concepts. This may be attributed to the use of 'Advance Organizer' in the former treatment. Marine (1977) found example only strategy superior to example-non-example strategy for identification of some concepts, but he himself expressed doubt about the validity of this finding. King (1976) obtained no significant differences between the three treatments on all the three criterion measures : knowledge, retention and heuristic transfer, but his treatments, as indicated earlier, deviated from the formats of the two models.

Content in Grabber's (1974) study was concepts in science. Marine (1977) and King (1976) used mathematical concepts as content but their subjects were seventh and fourth graders respectively.

2.8.0. Implications of Previous Research for the Present Study

1. Laboratory studies investigating relative effectiveness of expository and discovery methods used small samples, short time-spans and artificial conditions. They produced conflicting research evidence regarding efficacy of these methods for learning, transfer and retention of mathematical concepts. Since teaching of mathematics is invariably done in classrooms, dependable evidence for classroom applicability could be obtained through studies conducted within natural classroom settings only.

2. Classroom studies carried out in the past on this subject were fraught with conceptual issues, methodological problems and semantic inconsistencies in the uses of the word expository and discovery. Conclusive evidence, therefore, could not be produced from these studies. Hence, in a comparative study of expository and discovery approaches, formats of the two methods should be well defined, both methods should be

given fair treatment and important variables should be controlled.

3. Contribution of cognitive style to school learning has been observed in some studies. Studies exploring into the function of analytic-global dimension of cognitive style in learning mathematics seemed to have produced more consistent results than those involving field dependence-independence dimension. This suggested further exploration of conceptual style preference with regard to its effect on acquisition of mathematical concepts.

4. Although it has been claimed by the exponents of 'Advance Organizer Model', the studies reviewed did not seem to have indisputably confirmed the effectiveness of this model for learning and retention of mathematical concepts. Logical reasoning level and mathematical ability have been shown to be important factors affecting learning of mathematics by advance organizer treatment. This indicated greater applicability of the model for teaching mathematical concepts to formal operational students.

5. Although no investigator has employed full-fledged Bruner model as developed by Joyce and Weil, the studies concentrating on various facets of this model indicated its effectiveness for the teaching of mathematical concepts. Further, testing of the model for teaching formal operational students seemed warranted.

6. As indicated earlier, comparative studies involving some characteristics of Ausubel and Bruner models evidenced equal effectiveness for the teaching of mathematics. This, however, cannot be taken as full proof comparison between the two models since the results of piecemeal studies may not project the correct picture of the effectiveness of the models in their totality.

7. In a comparative study of the models of teaching all the characteristics of the models should be brought into play and teacher's fidelity to the prescribed models should be measured by suitable instrument.

8. From the studies reviewed, there appeared to be greater tendency towards using intact classrooms and teacher made instruments.

9. In most of the studies knowledge and retention were the criteria employed. Studies involving multiple criterion measures ought to be undertaken.

2.9.0. Rationale for the Study

The rational for undertaking this study is presented below :

1. It is realized that the problem of discovery and expository approaches for the teaching of mathematical concepts is still unsolved. Although Cronbach (1966) pointed out that "it is time to put aside the polemic question, Is teaching through discovery better than didactic teaching ?" Candor(1982) remarked that "the *distinction between reception and discovery instruction* is not one that psychologists appear *willing to lay to* rest". This assertion is further substantiated by large number of studies that have been carried out on this issue in recent years. The problem has sufficient pedagogical and theoretical importance and hence further attempt to resolve it seemed warranted.

2. Reviewers such as Wittrock (1966), Cronbach (1966), Herman (1969), Gustafsson (1975) and Turni, *et al.* (1977) have pointed out a number of inadequacies in previous research on this issue. Semantic inconsistencies in designating methods, unfair treatment to didactic group, methodological weaknesses, low technical quality, small samples, short time duration are the few to mention. Although increasing awareness about these deficiencies has been noticed and sporadic attempts to improve upon them have been observed in some of the recent studies, research evidence in this area is still inconclusive. One of the probable reasons for this state of affairs, according to the researcher, was a piecemeal approach adopted by researchers working in this field. An electic approach covering all important dimensions of expository and discovery presentations

was, therefore, expected to bring forth some new evidence in this area.

3. The models of teaching developed by Joyce and Weil (1972) made adoption of such an electic approach possible. Advance Organizer Model of David Ausubel stood for the choicest representative of expository approach and Concept Attainment Model of Jerome Bruner fully represented discovery approach. Moreover, since these models had well defined formats, direct comparison between them was expected to be relatively free from those impeding factors which vitiated previous research in this area.

4. Review of research revealed that there has not been any study directly comparing effectiveness of these two models for the acquisition of mathematical concepts even though Weil and Joyce (1978) suggested that "Advance Organizer Model and Concept Attainment Model are (mainly) concerned with acquisition and retention of concepts".

5. Research evidence with regard to efficacy of Advance Organizer Model is inconclusive : Barnes and Clauson (1975) concluded that "advance organizers as presently constructed generally do not facilitate learning". On the other hand, Luiten, Ames and Ackerson (1980) indicated that "average advance organizer study shows a small but facilitative effect on learning and retention." Present study hopes to throw further light on this contrasting evidence.

6. Although studies reviewed generally favoured Bruner model, Sheel (1981) remarked that, "...further research is needed to assess the impact of this (Bruner) strategy on student learning of concepts in the classroom environment". Present study ventures to satisfy this need to some extent.

7. There have been some efforts in India to train teachers for using these models in their day-to-day teaching. The Department of Education, Devy Ahilya Vidyapeeth,

Indore, recently (1982-83) conducted a series of training programmes for this purpose. In this context the researcher felt that these models be tested in Indian conditions before they are recommended to classroom teachers.

8. Cronbach and Snow (1977) state that aptitude treatment interactions do exist and that whereas no interactions have been confirmed well enough to be used as guides in making decisions about instruction, much has been learned. McLeod, *et al.* (1978) investigated relationship between field dependence/independence and expository versus discovery learning and showed that field independent perform best on working independently and field dependent perform best on providing extra guidance. This evidence, however, was not supported by the follow-up studies by McLeod and Adams (1979), Adams and McLeod (1979), and McLeod and Adams (1980). Exploration of some other dimension of cognitive style to study its differential effect was, therefore, thought necessary. Threadgill's (1977) investigation of the relationship between analytic-global cognitive style and didactic versus discovery method of instruction provided a thread for further exploration of this dimension of cognitive style. In the present study, therefore, conceptual style preference was chosen as a dimension of cognitive style to study the differential effect of categorical, descriptive and relational mode of conceptualization on the acquisition of mathematical concepts under the two strategies.

2.10.0. **Hypotheses**

On the basis of the review of research, following hypotheses, pertinent to objectives stated in section 1.7.0 , were formulated for being tested in the present study. For the convenience of subjecting them to statistical tests, the hypotheses have been stated in the null form.

To save space, following codes have been used in the statements of hypotheses and further in the analyses of data in Chapter IV.

The group taught by Ausubel Strategy :	A
The group taught by Bruner Strategy :	B
The Control group taught by Traditional Strategy :	C
Categorical Style subgroup of A	Ac
Descriptive Style subgroup of A	Ad
Relational Style subgroup of A	Ar
Categorical Style subgroup of B	Bc
Descriptive Style subgroup of B	Bd
Relational Style subgroup of B	Br
Categorical Style subgroup of B	Cc
Descriptive Style subgroup of C	Cd
Relational Style subgroup of C	Cr
Concept Knowledge Test	CKT
Concept Transfer Test	CTT
Heuristic Transfer Test	HTT
Short-term Retention Test	SRT
Long-term Retention Test	LRT

Hypotheses Pertaining To Objective I

I.1. There is no significant difference between the mean performance of A and C on CKT.

I.2. There is no significant difference between the mean performance of A and C on CTT.

I.3. There is no significant difference between the mean performance of A and C on HTT.

I.4. There is no significant difference between the mean performance of A and C on SRT.

I.5. There is no significant difference between the mean performance of A and C on LRT.

Hypotheses Pertaining To Objective II

II.1. There is no significant difference between the mean performance of B and C on CKT.

II.2. There is no significant difference between the mean performance of B and C on CTT.

II.3. There is no significant difference between the mean performance of B and C on HTT.

II.4. There is no significant difference between the mean performance of B and C on SRT.

II.5. There is no significant difference between the mean performance of B and C on LRT.

Hypotheses Pertaining to Objective III

III.1. There is no significant difference between the mean performance of A and B on CKT.

III.2. There is no significant difference between the mean performance of A and B on CTT.

III.3. There is no significant difference between the mean performance of A and B on HTT.

III.4. There is no significant difference between the mean performance of A and B on SRT.

III.5. There is no significant difference between the mean performance of A and B on LRT.

Hypotheses Pertaining to Objective *i*

i.c.1. There is no significant difference between the mean performance of Ac and Cc on CKT.

i.c.2. There is no significant difference between the mean performance of Ac and Cc on CTT.

i.c.3. There is no significant difference between the mean performance of Ac and Cc on HTT.

i.c.4. There is no significant difference between the mean performance of Ac and Cc on SRT.

i.c.5. There is no significant difference between the mean performance of Ac and Cc on LRT.

i.d.1. There is no significant difference between the mean performance of Ad and Cd on CKT.

i.d.2. There is no significant difference between the mean performance of Ad and Cd on CTT.

*i.d.*3. There is no significant difference between the mean performance of Ad and Cd on HTT.

*i.d.*4. There is no significant difference between the mean performance of Ad and Cd on SRT.

*i.d.*5. There is no significant difference between the mean performance of Ad and Cd on LRT.

*i.r.*1. There is no significant difference between the mean performance of Ar and Cr on CKT.

*i.r.*2. There is no significant difference between the mean performance of Ar and Cr on CTT.

*i.r.*3. There is no significant difference between the mean performance of Ar and Cr on HTT.

*i.r.*4. There is no significant difference between the mean performance of Ar and Cr on SRT.

*i.r.*5. There is no significant difference between the mean performance of Ar and Cr on LRT.

Hypotheses Pertaining to Objective *ii*

*ii.c.*1. There is no significant difference between the mean performance of Bc and Cc on CKT.

*ii.c.*2. There is no significant difference between the mean performance of Bc and Cc on CTT.

*ii.c.*3. There is no significant difference between the mean performance of Bc and Cc on HTT.

*ii.c.*4. There is no significant difference between the mean performance of Bc and Cc on SRT.

*ii.c.*5. There is no significant difference between the mean performance of Bc and Cc on CRT.

*ii.d.*1. There is no significant difference between the mean performance of Bd and Cd on CKT.

*ii.d.*2. There is no significant difference between the mean performance of Bd and Cd on CTT.

*ii.d.*3. There is no significant difference between the mean performance of Bd and Cd on HTT.

*ii.d.*4. There is no significant difference between the mean performance of Bd and Cd on SRT.

*ii.d.*5. There is no significant difference between the mean performance of Bd and Cd on LRT.

*ii.r.*1. There is no significant difference between the mean performance of Br and Cr on CKT.

*ii.r.*2. There is no significant difference between the mean performance of Br and Cr on CTT.

*ii.r.*3. There is no significant difference between the mean performance of Br and Cr on HTT.

*ii.r.*4. There is no significant difference between the mean performance of Br and Cr on SRT.

*ii.r.*5. There is no significant difference between the mean performance of Br and Cr on LRT.

Hypotheses Pertaining to Objective *iii*

*iii.c.*1. There is no significant difference between the mean performance of Ac and Bc on CKT.

*iii.c.*2. There is no significant difference between the mean performance of Ac and Bc on CTT.

*iii.c.*3. There is no significant difference between the mean performance of Ac and Bc on HTT.

*iii.c.*4. There is no significant difference between the mean performance of Ac and Bc on SRT.

*iii.c.*5. There is no significant difference between the mean performance of Ac and Bc on LRT.

*iii.d.*1. There is no significant difference between the mean performance of Ad and Bd on CKT.

*iii.d.*2. There is no significant difference between the mean performance of Ad and Bd on CTT.

*iii.d.*3. There is no significant difference between the mean performance of Ad and Bd on HTT.

*iii.d.*4. There is no significant difference between the mean performance of Ad and Bd on SRT.

*iii.d.*5. There is no significant difference between the mean performance of Ad and Bd on LRT.

*iii.r.*1. There is no significant difference between the mean performance of Ar and Br on CKT.

*iii.r.*2. There is no significant difference between the mean performance of Ar and Br on CTT.

*iii.r.*3. There is no significant difference between the mean performance of Ar and Br on HTT.

*iii.r.*4. There is no significant difference between the mean performance of Ar and Br on SRT.

*iii.r.*5. There is no significant difference between the mean performance of Ar and Br on LRT.

CHAPTER III

Design and Procedure

3.0.0. Prologue

Theoretical aspects considered in the first chapter and empirical evidence presented in the second chapter led to formulation of hypotheses. The third chapter is devoted to describing the research design that was planned and executed to test these hypotheses.

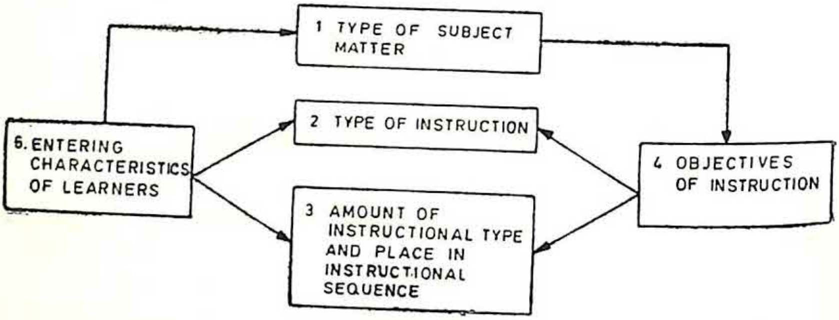
Kerlinger (1973, p. 300) defined research design as "the plan, structure, and strategy of investigation conceived so as to obtain answers to research questions and control variance." The plan includes an outline of what the investigator will do from writing the hypotheses to the final analysis of the data. The structure refers to scheme of the operation of variables. The strategy implies how the research objectives will be reached. By constructing an efficient research design the investigator attempts :

(1) to maximize the variance of the variables of his substantive research hypotheses;

(2) to control the variance of extraneous variables that may have effect on experimental outcomes; and

(3) to minimize the error variance.

Shulman (1970, p. 63) suggested a scheme for examining the variables which should be considered in formulating propositions about the best form of instruction, particularly when the practitioner is confronted with a contrasting array of positions. The scheme is depicted in Figure 3.1.


1 TYPE OF SUBJECT MATTER
2 TYPE OF INSTRUCTION
3 AMOUNT OF INSTRUCTIONAL TYPE AND PLACE IN INSTRUCTIONAL SEQUENCE
4 OBJECTIVES OF INSTRUCTION
5. ENTERING CHARACTERISTICS OF LEARNERS

Examples of each kind of variables

1. Mathematics, foreign language, social studies (subject matter defined in task terms).
2. Expository-discovery (degree of guidance); inductive-deductive.
3. Number of minutes or hours of instruction; position in sequence of instructional type.
4. Products; processes; attitudes; self-perception.
5. Prior knowledge; aptitude; cognitive style; values.

Fig. 3.1. Shulman Scheme for Examining the Variables.

Since Ausubel and Bruner models of teaching involved in the present study took contrasting positions with respect to attainment of educational objectives, the choice of variables for this study was done according to the Shulman scheme. The type of subject matter was mathematics. Type of instructions were Ausubel strategy and Bruner strategy. These two strategies included deductive expository and inductive discovery type of sequences respectively. Each type of instruction was imparted for twenty-five class periods of 40 minutes each. Objective of instruction was product centred, that is, acquisition of mathematical concepts. Entering characteristics of learners were previous knowledge, intelligence and cognitive style.

3.1.0. Experimental Variables

The variables chosen as per Shulman scheme were classified in the notation of experimental research. Operational definitions were generated for these experimental variables in order to add precision and avoid ambiguity.

3.1.1. Independent Variables

The independent variables in the present study were Ausubel Strategy, Bruner Strategy and Traditional Strategy in the form of control treatment.

That strategy in which the teacher

(1) begins his lesson by presenting an Advance Organizer;

(2) gives formal definition of the concept;

(3) illustrates the concept by examples;

(4) helps students assimilate the concept in their cognitive structure by relating it to the advance organizer;

(5) acts as primary source of knowledge; and

(6) retains control of the whole situation

was termed 'Ausubel Strategy'.

That strategy in which the teacher

(1) begins his lesson by presenting positive and negative examples of the concept;

(2) asks students to identify the concept by examining exemplars and non-exemplars;

(3) elicits formal definition of the concept from the students;

(4) makes students analyse their concept attainment strategies;

(5) poses not to be the primary source of knowledge; and

(6) remains liberal, allowing students to form their own hypotheses

was termed 'Bruner Strategy'.

That strategy in which the teacher

(1) begins his lesson by writing down definition of the concept on chalk-board;

(2) explains the definition by suitable examples;

(3) makes students commit the definition to memory by several repetitions;

(4) solves problems by mechanical application of definition;

(5) claims to be the primary source of knowledge; and

(6) behaves authoritatively

was termed 'Traditional Strategy'.

3.1.2. Dependent Variables

The dependent variables in the present study were knowledge, transfer, heuristic transfer, short-term retention and long-term retention.

Operationally, knowledge, transfer and heuristic transfer were defined as scores on Concept Knowledge Test, Concept Transfer Test and Heuristic Transfer Test respectively. Short-term retention and long-term retention were the scores on Concept Retention Test I and Concept Retention Test II respectively.

3.1.3. Moderator Variable

Conceptual style preference was taken as moderator variable in the present study.

Categorical style students were those who scored highest on the categorical response sub-test of the Cognitive Style Test.

Descriptive style students were those who scored highest on the descriptive response sub-test of the Cognitive Style Test.

Relational style students were those who scored highest on the relational response sub-test of the Cognitive Style Test.

3.1.4. Control Variables

Previous knowledge and intelligence were treated as control variables in this study.

Previous knowledge was operationally defined as a score on 'Previous Knowledge Test'.

Intelligence was operationally defined as a score on Raven's Standard Progressive Matrices Sets A, B, C, D, E.

3.2 0. Extraneous Variables and Their Control

Campbell and Stanley (1963, p. 175) have classified extraneous variables into two broad categories. Those that pose threat to internal validity include history, maturation, testing, instrumentation, selection, statistical regression and experimental mortality; and those that pose threat to external validity include artificiality of experimental setting, interference of prior treatment, contamination and Hawthorne effect. One important consideration while controlling extraneous variable is to maintain reasonable balance between internal and external validity of the experiment; for, if experimental controls are tightened to achieve internal validity, the more artificial and less realistic situation that may prevail reduces external validity or generalizability of the study. In the present study, an attempt has, therefore, been made to obtain a balance between the two.

3.2.1. Controls for Internal Validity

Following sources of extraneous variables were controlled to achieve internal validity.

History : In order to reduce the effect of history a control group was set up in this study and it was given the same material, the same homework and the same care as experimental groups.

Maturation : This was controlled by selecting experimental and control groups of nearly the same average age and giving them experimental treatment for exactly the same period of time (ten weeks).

Selection Bias : This was prevented by random assignment of treatments to groups which were fairly comparable on previous achievement in mathematics and further employing statistical controls at the stage of analysis.

Teacher Variability : All the three groups were taught by researcher himself to eliminate possible loss of interval validity due to teacher variability.

Non-adherence to Prescribed Treatments : This is a formidable source of internal invalidity and was taken care of by assessing teacher fidelity to prescribed treatments on the basis of specially trained observer raters' reports.

Unstable Instrumentation : The retest reliabilities of all the instruments used in this study were established prior to the experiment to prevent loss of internal validity due to unstable instruments.

3.2.2. Controls for External Validity

Following sources of external invalidity were controlled in this experiment :

Artificiality of Experimental Setting : In order to avoid this major threat to external validity the experiment was conducted in natural classroom setting. Controls employed to gain internal validity were such that they would not jeopardise the external validity of the experiment.

Contamination : Since the researcher was not associated with any of the experimental schools, contamination was out of question.

Contamination between Students of Experimental Groups : This was minimised by selecting experimental groups from different schools.

Howthorn Effect : According to Cook (1969, p. 204), "The Howthorn effect is a phenomenon characterized by an awareness on the part of the subjects of special treatment created by artificial experimental conditions." This awareness becomes confounded with the independent variable under study and subsequently leads to ambiguous results. In order to avoid this effect classroom conditions were maintained as natural as possible during the experiment and it was conceived from the subjects whether they were assigned to experimental treatment or control treatment.

3.3.0. Experimental Design

Selection of experimental design is naturally based upon the purposes of the experiment, the type of variables to be manipulated and the conditions or limiting factors under which it may be conducted.

Since applicability of the results of this study to classroom teaching of mathematical concepts was one of the prime concerns to the researcher, the experiment was conducted in natural classroom setting. The essential condition of a true experimental design, that of random selection of subjects to form equivalent treatment-groups, was difficult to be met in a classroom experiment. The principals of the schools would not allow their classes to be reshuffled. Hence, "The Non-equivalent Control Group Design" (Campbell and Stanley, 1963, p. 217) was chosen for the study. The design is diagrammatically presented in Figure 3.2.

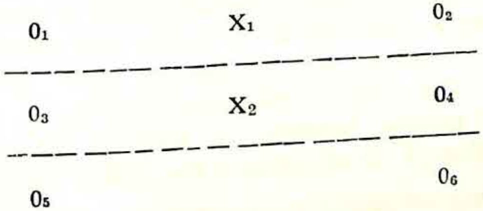


Fig. 3.2. Diagrammatic Presentation of the Non-Equivalent Control Group Design

Dashed line between the rows indicates non-equivalence of the groups. A blank in the third row represents control group receiving no experimental treatment but a place in the form of traditional treatment.

The procedures for this design are the same as for pretest-post-test control group design except that intact groups rather than randomly assigned ones are used, creating a control problem in terms of selection bias. This problem mandates the use of pretest measures to demonstrate initial equivalence of the intact groups on the dependent variable.

The use of 'Covariance Analysis' taking these pretest measures as covariates also serves the purpose. In addition, if the groups selected are as similar as possible on pre-treatment measures such as previous knowledge, intelligence, age and sex invalidity due to selection bias is further minimised. In the present study selection bias due to age and sex was controlled experimentally and that due to intelligence and previous knowledge was controlled statistically.

In respect of 'Non-equivalent Control Group Design' Kerlinger (1973, p. 342) states:

> The fact must be faced that frequently in research it is extremely difficult or impossible to equate groups by random selection or random assignment or by matching. Researchers commonly take pains to establish equivalence by other means and to the extent they are successful in doing so, to this extent the design is valid.

According to Campbell and Stanley (1963, p. 210), "Non-equivalent Control Group Design controls the main effects of history, maturation, testing, instrumentation, selection and mortality but fails to control internal invalidity due to regression and external invalidity due to interaction between selection and maturity, testing and treatment, and selection and treatment." The present researcher considered this to be a reasonable balance between the controls for internal and external validity of the experiment involved in this study.

The purpose of including moderator variable—Conceptual Style Preference—in this study was only to ascertain how it modifies the relationship between independent and dependent variables. This could be achieved by carrying out separate analyses of covariance for the subgroups of each of the three groups showing particular style preference. The study of interaction was kept beyond the purview of this study since the researcher visualised in advance the difficulties in arranging factorial experiment in classroom setting.

3.4.0. Selection of Subjects

Since pre-treatment measures of previous knowledge and intelligence were to be used as constant covariates in the analysis of all dependent measures, preliminary selection of treatment groups from intact classes was done on the basis of size of the class, average age of students in the class and means and standard deviations of the marks in mathematics obtained by the students in Secondary School Certificate Examination. Randomization technique was used at all stages where it was possible.

3.4.1. Selection of School Classes

The principals of all the eighteen Higher Secondary Schools running eleventh class of science stream during the academic

TABLE 3.1

Summary of the Data for Preliminary Selection of Schools

School	1	2	3	4	5	6	7
No. of Students	37	44	42	43	42	50	51
Average Age	16.57	15.84	15.39	15.66	15.07	16.73	14 84
Mean Score in Maths.	48.34	59.67	54.93	55.80	56.67	49.58	74.20
Standard Deviation	13.31	12.00	12.04	13.05	13.92	15.81	14.89

session 1981-82 were contacted and seven schools were randomly chosen from these to record data for the preliminary selection. Descriptive statistics of mean and standard deviation of the marks obtained by the students in each of these schools is presented in Table 3.1.

On the basis of this data, schools 2, 3, 4 and 5 were provisionally selected since the classes in these schools were nearly similar in respect of size, average age, mean score and standard deviation of S.S.C. marks in mathematics.

3.4.2. Allotment of Treatments

Random assignment of treatments to classes was done as follows:

The teachers of schools 2, 3, 4, 5 were designated by symbols P, Q, R, S and the three treatments Ausubel, Bruner and control by A, B, C respectively. Four sets of three slips each, bearing the letters A, B, C, were prepared. P, Q, R, S were requested to draw one slip from a set given to each of them. The outcomes were as given in Figure 3.3.

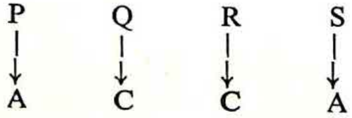


Fig. 3.3. Initial Outcomes in Random Assignments of Treatments to Classes.

Since P and S both drew A, a coin was flipped and A was allotted to S who won the toss. Similarly, C was allotted to R by toss. B was to be allotted either to P or Q who lost the first toss. Finally, it was allotted to Q who won the second toss. The school classes of Q, R and S, thus, became treatment groups for Bruner, Control and Ausubel strategies respectively. The school class of P could not be included in the study but was used for finding test-retest reliabilities of the instruments. Preliminary data for the three treatment groups has been separated out in Table 3.2.

3.4.3. Final Sample

Although the experiment started with treatment groups

TABLE 3.2

Summary of Preliminary Data for the Three Schools Selected for Experimental Teaching

Group	Size	Average Age	Mean	S.D.
Ausubel	42	15.07	56.67	13.92
Bruner	42	15.39	54.93	12.04
Control	43	15.66	55.80	13.05

almost equivalent in respect of size, average age, mean score in mathematics at S.S.C. and variability of these scores, each group lost some subjects during experiment due to various reasons such as transfer from the school, absence at one or more of the tests, or attendance below ninety per cent in experimental session. Final analysis, therefore, had to be restricted only to those subjects who survived the mortality. Data for these final groups has been presented in Table 3.3.

TABLE 3.3

Summary of Preliminary Data for the Final Sample of Subjects

Group	Size	Average Age	Mean	S.D.
Ausubel	37	15.03	57.67	13.89
Bruner	36	15.42	54.23	12.33
Control	38	15.65	56.20	13.12

It can be observed from Table 3.3 that the treatment groups were still fairly comparable on these pre-experimental measures.

3.5.0. Choice of Content

Content was chosen from the syllabus of mathematics prescribed by Maharashtra State Board of Secondary Education for eleventh grade students of science stream. Taking into account the nature of teaching strategies involved in this study, the concepts of set, relation, function, limit, continuity and derivative were selected in consultation with subject teachers of the experimental classes. These concepts were hierarchically connected with each other and each of them were branched into many other subordinate concepts as shown in Appendix A. This facilitated planning of lessons by Advance Organizer Model using expository organizers for the major concepts and comparative organizers for subordinate concepts. Moreover, since each of the concepts had well defined attributes, planning of lessons by Concept Attainment Model was also possible.

3.6.0 Lesson Plans

The subject matter to be covered was split up into twenty-five units involving a concept or its subordinate as shown in Appendix B. A guiding principle in the choice of unit was its suitability for being taught by both the experimental strategies. Three lesson plans were prepared on each unit—one according to Ausubel strategy, one according to Bruner strategy and one according to traditional strategy. Formats of the lesson plan by these three strategies are given in following three subsections. Detailed lesson plans on Ausubel, Bruner and Traditional strategies have been displayed in Appendices C, D, E respectively.

3.6.1. Format of Ausubel Lesson Plan

Lesson plans pertaining to Ausubel strategy included the following steps adapted from "Planning Guide for the Advance Organizer Model" (Weil and Joyce, 1978, pp. 256-257).

I. Advance Organizer
    1. Essential Characteristics of the Advance Organizer
    2. Type of Advance Organizer : Expository/Comparative
    3. Formulation of Advance Organizer Presentation

II. Statement of Behavioural Objectives

III. Presentation of Learning Task.

IV. Strengthening Cognitive Structure.

V. Description of Teacher's Activities in each phase of the strategy.

3.6.2. Format of Bruner Lesson Plan

Lesson plans pertaining to Bruner strategy included the following steps adapted from "Planning Guide for Concept Attainment Model" (Weil and Joyce, 1978, pp. 89-90) :

I. Analysis of Concepts :
   1. Name of the concept.
   2. Attributes of the concept.
   3. Type of concept : Conjunctive/Disjunctive/Relational.

II. Statement of Behavioural Objectives.

III. Presentation of Exemplars and Non-exemplars.

IV. Concept Attainment Strategies.

V. Description of opening moves of the teacher in each phase of the strategy.

3.6.3. Format of Traditional Lesson Plan

Lesson plans pertaining to traditional strategy involved the following steps which represented placebos for steps included in experimental strategies :

I. Introduction :
   1. Recapitulation of previous knowledge.
   2. Naming the concept.
   3. Importance of the concept.

II. Statement of Behavioural Objectives.

III. Presentation of Definition.

IV. Exemplification of Definition.

V. Description of Teacher's Verbalization in each phase of the strategy.

3.7.0. Instrumentation

In order to draw any valid conclusion from an experimental research, instruments used for the measurement of variables should be reliable and valid. This requirement is usually met by employing commercially available standardized tests. Subject matter involved in the present study was, however, such that standardized tests were not available and number of tests had to be constructed before starting the experiment.

Intelligence was measured by Raven's Standard Progressive Matrices. For the measurement of cognitive style, however, Robinson's test had to be suitably adapted. All other instruments required for the measurement of previous knowledge, concept knowledge, concept transfer, heuristic transfer, short-term retention and long-term retention were developed by the researcher.

In constructing these tests the procedure suggested by Tuckman (1972, pp. 164-166; 1975, pp. 67-122) and Anastasi (1961, pp. 428-436) was adopted.

According to this procedure test construction begins as a first step with the preparation of Content Outline—a list of concepts or tasks to be mastered, each prefixed by action verb to indicate instructional objective. The second step is to prepare a pool of items on the basis of content outline. The third step is item-analysis to determine the indices of difficulty and discriminability of each item in the pool. The fourth step is to assemble items with satisfactory discriminability (above .67) and reasonable difficulty (between .33 and .67) in proportion of predetermined weightages.

The test constructed in this manner has in-built content validity which ensures reliability in the form of internal consistency. External consistency or stability of the test could be determined by computing test-retest reliability.

According to Jensen (1980, p. 262), a desirable feature of an ability test is that "the test is short, consisting of just a few items so that little testing time is required." A reasonable limit was, therefore, put on the length of all the tests developed in this study.

3.7.1. **Raven's Standard Progressive Matrices**

One of the most widely used tests for the measurement of intelligence is Raven's Standard Progressive Matrices Sets A, B, C, D and E. This test was particularly suitable for the present study since it measures the ability of observation and clear thinking. Bruner (1956, p. 246) asserts, "Concept attainment is, to be sure, an aspect of what is conventionally called thinking...."

The test consists of sixty matrices, or designs, from each of which a part has been removed. The subject chooses the missing insert from six or eight given alternatives. The items are grouped into five series, each containing twelve matrices of increasing difficulty but similar in principle. The earlier series require accuracy of discrimination; the latter, more difficult series involve analogies, permutation and alteration of pattern, and other logical relations.

It is a group test and is culture free. Moreover, it is a power test in a sense that it does not stipulate any time limit for completion by the students. Retest reliability of the test, according to Raven (1960, p. 2), varies with age, from 0.83 to 0.93. For the age group of students in this study it was 0.93. Concurrent validity of the test with Terman-Merril Scale has been given to be 0.86.

3.7.2. Cognitive Style Test

As indicated earlier, the dimension of cognitive style used in this study was 'Conceptual Style Preference.' Kagan, Moss and Sigel (1963) developed conceptual style test consisting of thirty trials of pictures to identify conceptual style preference. Brozobich, Hall and Watson (1972); Scott (1971) and Wallach and Kogan (1965), however, found that the scoring based on this test were irrelevant for school learning tasks. Robinson and Gray (1974, p. 794) developed three separate scales, each containing fourteen analogy items, to identify conceptual style preference of the subjects. They argued that verbal analogy format provided means of assessing cognitive style as a process variable since a subject must form relationship between the first two terms and then generalize the relationship between the first two terms to the last two terms of the analogy. Using this test the authors found that additional variance was accounted for by cognitive style measure beyond that of verbal and non-verbal

I.Q. in relation to school learning. This test, however, was culturally biased since the items in the test were chosen to suit the sample of students from Midwestern City. The researcher, therefore, adapted the format of this test to develop cognitive style test suitable for the subjects involved in the present study.

The test consisted of three different scales for the three styles of conceptualization. Each scale contained fifteen verbal analogy items arranged in increasing order to complexity. The subject had to identify a concept or a relation between the first two terms and then associate the third term with one of the three choices given for the fourth term so as to obtain similar concept or relation between them. That scale on which the subject scored highest indicated his conceptual style preference.

Concurrent validity of the three scales with those of Robinson and Gray were found to be .523 for categorical style, .618 for descriptive style and .535 for the relational style (N=42). Test-retest reliabilities of these three scales were found to be .560, .626 and .527 (N=37) respectively. The test and its scoring key has been given in Appendix F.

3.7.3. Previous Knowledge Test

This test was assigned to assess the knowledge of concepts learnt by the students in previous two grades. The test consists of twelve multiple choice items on the content outline suggested by the subject teachers of experimental classes. Fixed choice format was chosen since recognition of concepts was considered enough for the purpose. The test had content validity by virtue of its construction. Test-retest reliability was found to be .574 (N=44). The test is given in Appendix G of this report.

3.7.4. Concept Knowledge Test

This test was designed to measure knowledge of the concepts taught during experimental teaching at fixed intervals. The test consisting of ten items is divided into three parts. The first part on the concepts of set, relation and function contains four items; the second on the concepts of limit and continuity contains four items; and the third on the concept of derivative contains two items. These three parts were to be administered after the completion of respective topics. The scores on the three parts were then to be pooled together to obtain a composite score on the criterion of concept knowledge.

Fixed choice format was chosen for the test since it purported measurement of the recognition of concepts. Multiple choice items were preferred since they reduce the chances of correct guessing.

The test had content validity since it included all the major concepts or their subordinate concepts on the basis of preplanned content outline. Test-retest reliability determined by repeated administration to the same twelfth class students after fifteen days was found to be .802 (N=39). The test is given in Appendix H of this report.

3.7.5. Concept Transfer Test

This test was designed to measure the ability of students to transfer the knowledge of concepts gained during experimental teaching. According to Bruner (1957), transfer means "going beyond the information given." In the present study transfer meant application and extrapolation of knowledge in the situation similar to that presented during teaching. Free choice format was preferred since transfer necessarily required recall of the concepts.

Total number of items in the test was ten. Depending upon the concepts involved in them four of these items were included in the first part, four in the second part and two in the third part of the test. The score on concept transfer test was to be obtained by pooling together the scores on these three parts administered immediately on completion of units involved in them.

The test had in-built content validity by virtue of its construction on the basis of content outline prepared in advance. Test-retest reliability was determined by repeated administration to the same twelfth grade students after fifteen days and was found to be .554 (N=41). The test is given in Appendix I.

3.7.6. Heuristic Transfer Test

Learning the heuristics, according to Bruner (1961), is the learning of attitude and activities that go with inquiry and research and is only achieved through the act or problem-solving and the effort of discovery. In the present study, heuristic transfer meant application, analysis and synthesis of the concepts learnt to discover new relationships in situations different from those presented during teaching.

Essay items were chosen for they test the ability of students to think out and use what they know. According to Tuckman (1975, p. 111), "Essay items provide test-takers with the opportunity to structure and compose their own responses within relatively broad limits." Such an opportunity is evidently essential for heuristic transfer.

Total number of items in the test was ten. On the basis of content involved, four of them were included in the first part, four in the second part and two in the third part. Total score on heuristic transfer was to be obtained by pooling together the scores on these three parts administered on completion of units involved in them.

The test had content validity since it was based on previously prepared content outline. Test-retest reliability was determined by double administration with a time gap of fifteen days to the same group of twelfth grade students. It was found to be .821 (N=33). An elaborate marking scheme was also prepared to arrest scorer's bias. The test is presented in Appendix J of this report.

3.7.7. Concept Retention Test—I

This test was designed to measure retention of concepts taught after two weeks from the completion of teaching. Multiple choice items left over from the pool for concept knowledge test were transformed into free-choice fill-in type. Free choice format was preferred since the main purpose of the test was to test recall. There were ten items in the test to be administered at a time and completed within twenty minutes. Content validity was ensured by including all the major concepts taught during experimental teaching. Test-retest reliability was found to be .606 (N=27). The test is presented in Appendix K.

3.7.8. Concept Retention Test—II

This test was designed to measure retention of concepts taught after eight weeks from the completion of teaching. Free-choice fill-in type items were those left over from the pool prepared for concept knowledge test after taking the items for Concept Retention Test—I. There were ten items to be administered at a time and completed within twenty minutes. Items for all the major concepts taught were included to ensure content validity. Test-retest reliability was found to be .411 (N=32). The test is presented in Appendix L.

3.7.9. Observer's Rating Scale

This scale was designed to measure teacher fidelity to prescribed strategy during experimental teaching. The degree to which the teacher adheres to a given treatment is reflected in the occurrence of teaching behaviours that characterize the treatment. Necessary data for the measurement of teacher fidelity can, therefore, be obtained by careful observation and rating of teaching behaviours on a scale.

Worthen (1970) developed a scale to rate teaching behaviours through classroom observation. The items on his scale, however, could discriminate only between expository and discovery treatments used in his study. The format of this scale was, therefore, adapted to construct a new scale which could serve to discriminate between the treatments employed in the present study.

Major items along which the discrimination could be done were identified to be as follows :

A. Method of Induction.
B. Introduction of Generalization.
C. Mode of Concept Acquisition Evoked.
D. Interjection of Teacher Knowledge.
E. Degree of Structure.

For each of these items, five characteristics of teacher behaviour were delineated as per following correspondence :

1 ↔ perfect adherence to Bruner strategy

2 ↔ incomplete adherence to Bruner strategy.

3 ↔ non-adherence to either strategies.

4 ↔ incomplete adherence to Ausubel strategy.

5 ↔ perfect adherence to Ausubel strategy.

Each item of the lesson was to be rated by encircling one number from 1 through 5, depending upon the characteristic observed. Total absence of the item or failure to observe it was to be shown by encircling 0. Average of the five ratings for five items could be used to assess teacher fidelity.

Perfect average rating for Bruner strategy would be '1' and that for Ausubel strategy would be '5'. The teacher could be said to have adhered to Bruner strategy to the extent the average rating of his lesson approximates '1' and to the Ausubel strategy to the extent the average rating of his lesson approximates '5'. The extent to which the average rating deviates from '1' or '5' indicates the deviation from Bruner and Ausubel strategies respectively. Since the control group treatment did not possess characteristics of either of the two strategies, average rating for this treatment should approximate the middle point of the scale.

Teacher fidelity to Bruner strategy would, therefore, be indicated by insignificant difference between obtained average rating and perfect average rating for this strategy, that is, 1. Similarly, teacher fidelity to Ausubel strategy would be indicated by insignificant difference between obtained average rating and perfect average rating for this strategy, that is, 5.

The scale and the continuum of characteristics under each item are given in Appendix M.

3.8.0. Observer-Raters

Six observer-raters were selected for observation and rating of experimental lessons. Three of them were internal and three external. One week training programme was organized during winter vacation of 1981 to orient the observer-raters in background knowledge required for the proper conduct of observation and rating. Since all of them were highly qualified persons, the training was informal. It began with brief discussion of subject matter involved in the study. Major concepts to be taught were fully analysed. The central topic in the training programme was the models of teaching. Ausubel's Advance Organizer Model and Bruner's Concept Attainment Model were explained in detail. The bearing of these models to experimental strategies was made explicit. Phases involved in each strategy and teacher's role in each phase of the strategy were clarified. Formats of the lesson plans for each strategy were presented and some lesson plans were scrutinised. Finally, each observer-rater was given a form of observer's rating scale with full instructions, and continuum of characteristics delineated under each item was explicated. Doubts and difficulties raised by some of the observer-raters were clarified.

Internal observers were allotted their own schools for the entire period of experimental teaching. External observers, however, were rotated and required to visit different schools as per programme prepared in advance. Each lesson of two experimental groups and a control group was to be observed by two observer-raters, one internal and one external.

3.9.0. Pretesting

The experiment started immediately on reopening of schools after winter vacation of 1981. On the very first day the researcher visited all the three schools and collected attendance registers of the classes from the principals. He, then, went to the classes with subject teachers. On having been introduced by the subject teacher he spent some time in establishing rapport with the students, while taking their roll call. The purpose of the experiment was then briefly explained to the students, concealing the nature of treatment they were going to be given. The students were told that the researcher would be taking three periods of mathematics per week allotted to him by the principal of their school and that he would continue to teach for nearly two months covering major topics in their syllabus. In order to observe the effect of his teaching he would like to know their present position; for which they would be given some tests in the following three periods which they must not miss.

The periods allotted to the researcher fell on different days and at different times. Hence, it was not possible to administer a given test to all the three classes on the same day or at the same time of the day.

Cognitive Style Test, Raven's Standard Progressive Matrices Sets A, B, C, D, E, and Previous Knowledge Test were administered to the three classes in the same order during the allotted periods.

While administering cognitive style test and previous knowledge test no oral instructions in the beginning were given since they were printed on the question-answer forms themselves. Each of these tests could be completed in one class period.

While administering Raven's Standard Progressive Matrices Test, instructions contained in the Guide to the test were strictly adhered to. It was made sure that each student followed the procedure of doing the test. Students were told that they could take as much time as they wanted to complete the test. Teachers of next following periods were requested to spare their periods. Answer-sheet and a booklet were collected from the student only when he has completed it or said to have completed it.

During testing, seating arrangement in each class was such that students did not get any chance to copy answers from the neighbouring students. Subject teachers of the classes assisted the researcher in keeping strict supervision.

3.10.0. The Conduct of Experiment

Experimental teaching commenced on 12th November, 1981, and lasted for ten weeks till 20th January, 1982. This particular period was purposefully chosen for the following reasons:

(1) It was preceded by a fortnight's holiday, minimizing, thereby, the possible carry-over effect of previous instruction.

(2) Students were in a jubilant mood, having enjoyed a festive season of Dipavali.

(3) There were no school examinations during this period to cause undesired anxiety to students.

(4) Climatic conditions of the city were best during this season and hence precluded any disruption in the programme due to unforeseen natural conditions.

The researcher himself was to teach all the three classes. Time-table was, therefore, so arranged that periods of two experimental groups should not fall on the same day in order to avoid contamination between the two experimental strategies. The time of the day for teaching these two groups, however, was arranged as close as possible. The periods of Ausubel group were taken on Thursday, Friday and Saturday from 2.45 p.m. to 3.25 p.m. and those of Bruner group were taken on Monday, Tuesday and Wednesday from 2.05 p.m. to 2.45 p.m. These were the first periods after long recess in the respective schools. The control group was taught on Monday, Tuesday and Wednesday after a period of short recess in the school from 1 .20 p.m. to 1.00 p.m.

Each student was provided with a notebook to be used exclusively for taking down notes of the lessons taken by the researcher. Each lesson started with a roll-call and closed with assignment of home work, which was the same for all the three groups.

Teaching was done strictly according to lesson plans drawn in advance as per requirement of the three strategies. Normally, the units were completed in one class period. In the few cases where it could not be possible due to unexpected questions from the students, which were respected and promptly responded to, the remaining portion was covered in the next ensuing periods. If their blooming faces were to be any indication, the students of all the three groups enjoyed the lessons. The last bell often came as surprise to them.

It was a great satisfaction to the researcher that observers were present during all the lessons of all the three strategies. Their presence could not be a source of distraction to the students since they used to be seated on the back benches of the classes Nor could it inhibit the researcher in any way since he was accustomed to being observed while teaching. Observer's reports were collected immediately after the lesson and specific suggestions, if any, were incorporated in the conduct of next lessons.

There was no discontinuity in the programme except that lessons of B and C classes on 4th, 5th and 6th January had to be postponed due to a cricket match between their schools.

3.11.0 Post-Testing

Criterion tests I, II and III were administered to all the three groups in the periods following the completion of eighth, twentieth and twenty-fifth units of experimental teaching respectively.

Criterion test I consisted of the first parts of concept knowledge test, concept transfer test and heuristic transfer test. Criterion test II consisted of the second parts of these tests and Criterion test III consisted of the third parts of the same tests. Criterion tests I and II were of thirty-two minutes' duration since they contained four knowledge items of two minutes each, four transfer items of two minutes each, and four heuristic transfer items of four minutes each. Criterion test III was of sixteen minutes' duration since it contained two knowledge items of two minutes each, two transfer items of two minutes each, and two heuristic transfer items of four minutes each.

Students were informed about testing only a day earlier. They were also told that they need not cram anything as a part of preparation for the test since the questions would test their conceptual understanding rather than rote-memory. Subject teachers were requested to make proper seating arrangement for the test which they did without fail.

Cyclostyled question papers were distributed at the first stroke of the bell. Answers were to be written in the spaces provided on the question paper itself. All instructions were furnished in writing on the question paper. Three minutes were given to read the instructions and to complete initial entries. Students started writing the answers at the second stroke of the bell. Every precaution was taken that there would be no outside disturbance during testing. Subject teachers helped the researcher in keeping strict supervision. There were no malpractices of any kind. Warning bell was rung two minutes prior to closing time. Students stopped writing at the fourth stroke of the bell. Papers were collected immediately.

3.11.1. Administration of Retention Tests

Retention test I was administered to all the three groups exactly after two weeks from the completion of experimental teaching. Group A took the test on 1st February, 1982, and groups B and C on 4th February, 1982.

Retention test II was administered to all the three groups exactly after eight weeks from the completion of experimental teaching. Group A took it on 15th March, 1982, and groups B and C on 18th March, 1982.

Students were not informed about the dates and timings of these tests. Subject teachers were, however, requested a day before to make provision of proper seating arrangement. Both the tests were of twenty minutes' duration and were completed within half of the period after which the classes were engaged by their subject teachers.

3.12.0. Scoring

The answer sheets of students in the three groups for all the pre- and post-tests were scored independently by the researcher and his senior colleague who was also one of the observer-raters.

Answer sheets of Progressive Matrices were scored using the scoring key provided with "Guide to the Standard Progressive Matrices Sets A, B, C, D and E" (Raven, 1960). Scoring key prepared by the researcher for cognitive style test was used in scoring the answer sheets of Cognitive Style Test.

The correct answers to all objective type questions in Previous Knowledge Test, Concept Knowledge Test, Concept Transfer Test and the two retention tests were first noted down and then compared with those of the students while scoring these tests. Special care was taken while scoring essay type items on Heuristic Transfer Test. Marking scheme prepared in advance by the researcher was strictly adhered to by both the scorers.

There was no disparity between the scores allotted by the two scorers in respect of progressive matrices and cognitive style tests of all the three groups. A few discrepancies due to oversight were identified in the scoring of objective tests by the two scorers which were consequently corrected. While scoring heuristic transfer test the two scorers differed in allotting marks to some items in spite of following the marking schemes. Average of the two scores was assigned to the item in five such cases; two from group A and three from group C.

Since the scores allotted by the two scorers were thoroughly checked and corrected or averaged as the case may be, the researcher did not think it necessary to compute the coefficient of correlation between the scores allotted by the two scorers in order to establish the reliability of scoring.

3.13.0. **Inter-Observer Reliability**

As has been indicated earlier in section 3.8.0., each lesson of the three strategies in this study was observed by two observer-raters, one internal and one external. In order to assess the accuracy of these ratings for their further use in the measurement of teacher fidelity, it would be necessary to find inter-observer reliability.

Medley and Mitzel (1963, p. 253) discussed three types of coefficients for the reliability of observation by any observational technique.

(*i*) *Reliability Coefficient* : A correlation between the scores based on observations made by different observers at different times.

(*ii*) *Stability Coefficient* : A correlation between scores based on observations made by the same observer at different times.

(*iii*) *Coefficient of Observer Agreement* : A correlation between scores based on observations made by different observers at the same time.

According to these authors, the coefficient of observer agreement tells us something about the objectivity of an observational technique; the coefficient of stability tells us something about the consistency of behaviour from time to time. But only the reliability coefficient tells us how accurate our measurements are. In order to find inter-observer reliability, for the purpose of this study, reliability coefficient was, therefore, the appropriate type among the three. It was determined as follows :

The scores given by the two observers while simultaneously observing the different lessons of the same strategy were first calculated by averaging their ratings for five items of the scale in respect of each lesson. Rank-Difference Coefficient of correlation was then computed between the two sets of scores corresponding to two observers. This was done for each different pairs of observers.

For the sake of brevity internal observers for Ausubel, Bruner and Control strategies have been designated by I_1, I_2 and I_3 respectively and the three external observers by E_1, E_2, and E_3. As E_1, E_2, and E_3 were rotated among the three groups, each of I_1, I_2, and I_3 was paired with E_1, E_2 and E_3 in observing nearly equal number of lessons of the strategy. Table 3.7 shows the pairs of observers observing the lessons of each strategy, number of lessons observed by each pair and also the inter-observer reliability for each pair.

These high values of inter-observer reliabilities for all pairs of observers, except possibly one (I_3 E_2), were taken as an adequate evidence of the accuracy of ratings given by different observer-raters. Relatively low value of inter-observer reliability for the pair I_3 E_2 could be attributed to the fact that one of the observers came late for two lessons and consequently scored some items as zero since he could not observe them.

3.14.0 Statistical Technique for Data Analysis

The data collected by administration of pre- and post-test measures to the three groups was analysed using appropriate statistical technique. The analyses have been presented in the next chapter.

Descriptive statistics used for summarizing the data were means and standard deviations. Although groups were equated

TABLE 3.4

Inter-Observer Rehability for Each Pair of Observers, Observing the Lessons of the Three Strategies

Strategy	Pairs of Observers	No. of Lessons Observed	Inter-Observer Reliability
A	I_1 E_1	8	.94
	I_1 E_2	8	.93
	I_1 E_3	9	.94
B	I_2 E_1	8	.90
	I_2 E_2	8	.98
	I_2 E_3	9	.96
C	I_3 E_1	9	.91
	I_3 E_2	8	.83
	I_3 E_3	8	.94

on the basis of means and standard deviations of the marks obtained in mathematics in S.S.C. examination, the possibility of their differences on pretreatment measures—previous knowledge of the concepts and intelligence—could not be ruled out. This was, precisely, the reason for the choice of "Non-Equivalent Control Group Design" for the study. As per requirement of this experimental design, pre-treatment differences between experimental and control groups must be controlled either experimentally or statistically. Experimental control of these variables was not possible, since the study was conducted with intact classes as experimental groups. Imposing statistical control was, thus, inevitable. Natural choice of statistical technique for the data analysis in this study was, therefore, "Analysis of Covariance".

Analysis of covariance is an extension of the analysis of variance combined with certain features of regression analysis and allows the researcher to statistically equate the independent variable groups with respect to one or more variables which are relevant to the dependent variable.

Requisite assumptions for the proper interpretation of analysis of covariance results are those of both linear regresssion and analysis of variance. From regression, it must be assumed that:

(1) the relationship between independent and dependent variables is linear;

(2) homoscedasticity exists, that is, variances in the values of dependent variable for any given values of independent variable are comparable; and

(3) distribution of dependent variable for given values of independent variable is approximately normal.

Analysis of variance dictates that:

(4) measures within each group must be randomly drawn; and

(5) variances within the groups must be relatively homogeneous.

Popham (1967, p. 230) points out that, "...stringent satisfaction of these assumptions is probably not required, but the departure from them should not be too great". McNemar (1962, p. 252) asserts:

> Although these assumptions are incorporated in the mathematical derivation of the F distribution, there is ample evidence that marked skewness, departure from normal kurtosis, and extreme differences in variance do not greatly disrupt the F test as a basis for judging significance in the analysis of variance.

The assumption of 'Homogeneity of Variance' was, however, important since extension of 't' test for more than two samples —as is done in analysis of variance or for that matter, in analysis of covariance—required equality of variance for its valid use. Bartlett's test (Snedecor and Cochran, 1967, p. 296) was, therefore, employed to check the homogeneity of variance before proceeding to analysis of covariance for each set of data.

Intelligence was treated as constant covariate in the data analysis of all criterion measures by employing "Analysis of Covariance" technique. Another covariate used while analysing data on "Concept Knowledge Test" was previous knowledge. In the data analysis of other criterion measures, however, thecovariate 'Previous Knowledge' was replaced by 'Concept Knowledge' following Worthen (1970). While analysing data on "Long-term Retention Test", 'Short-term Retention' was taken as an additional covariate since the two measures were functionally related.

In case significant mean-differences among the groups on a criterion measure were indicated by F-test, further analysis for the identification of groups between which the mean-differences actually existed was done by employing 't' test.

3.14.1. Levels of Significance

Hypotheses were tested against a probability level of .01 to prevent Type I error in drawing inferences. Rejection of null-hypothesis when it was true was considered to be more serious an error than accepting the null-hypothesis when it was false, since, in the present study, comparison was done between the two models of teaching which were giants in themselves. Moreover, previous research, reviewed earlier, indicated the possibility of these two models having equal effectiveness with regard to some criteria used in this study.

However, care was taken to avoid the occurrence of Type II error as well. Those hypotheses which could have been accepted at .01 level of significance were tested at .05 level of significance.

No directional prediction in respect of mean difference on any criterion was done in this study. Hence, two-tailed test of significance was used throughout.

CHAPTER IV

Analyses and Results

4.0.0. Prologue

Of utmost importance in the experimental research on comparison of instructional treatments, as pointed out by Medley and Mitzel (1963, p. 249), is "assessment of the degree to which the methods supposed to be applied were actually applied." Measurement of teacher fidelity to prescribed treatments was, therefore, considered as a prerequisite to analyses and interpretations of the data on criterion measures in this study.

Consequently, this fourth chapter begins with analysis of data on Observer's Rating Scale to bring forth evidence of teacher's adherence to prescribed teaching strategies in this study. This follows presentation of summarized data on pre-treatment measures. Descriptive and inferential analyses of the data on criterion measures have, then, been serially presented. This involved criterion-wise testing of hypotheses using 'Analysis of Covariance' and subsequent employment of 't' test where necessary.

Analyses for testing minor hypotheses (hypotheses pertaining to subsidiary objectives) have been described immediately after presentation of the test of major hypothesis (hypothesis pertaining to principal objective) which covers them.

Inferences have been drawn from each analysis and the results of all analyses have been summarized at the end of the chapter in section 4.16.0.

4.1.0 Assessment of Teacher Fidelity

In order to assess teacher fidelity to prescribed teaching strategies, Worthen's (1970) procedure was adopted for analysing the data on 'Observer's Rating Scale'.

Average score for each lesson was first obtained by taking the mean of scores allotted to that lesson by the two observers, observing the lesson simultaneously. Mean score assigned by each pair of observers to a particular strategy was then computed by taking the mean of average scores of the lessons of that strategy observed by that pair of observers. Finally, the weighted mean of the mean scores assigned by three pairs of observers to that strategy was calculated to obtain grand score for that strategy. Table 4.1 shows the mean scores assigned by each pair of observers to a specific strategy and grand score computed therefrom for the three strategies.

TABLE 4.1

Grand Scores for the Three Strategies Obtained on the Basis of Observer Raters' Reports

Strategy	Pair of Observer Raters	No. of Lessons Observed	Mean Score	Grand Score
A	I_1E_1	8	4.263	4.628
	I_1E_2	8	4.788	
	I_1E_3	9	4.700	
B	I_2E_1	8	1.388	1.364
	I_2E_2	8	1.363	
	I_2E_3	9	1.344	
C	I_3E_1	9	2.233	2.172
	I_3E_2	8	2.463	
	I_3E_3	8	1.813	

The grand score for each strategy was compared with theoretically perfect score for that strategy in order to assess the proximity between the two. Table 4.2 shows 't' values for differences between grand score and perfect score for each strategy.

TABLE 4.2

Significance of Differences Between the Grand Score and Perfect Score for the Three Strategies

Strategy	Grand Mean M_1	S.D. σM_1	Perfect Mean M_2	S.D. σM_2	Diff. M_1-M_2	S.E.σ. σD	t
A	4.628	.699	5	.807	.372	.213	1.754
B	1.364	.350	1	.551	.364	.131	2.756
C	2.172	.568	3	1.046	.828	.237	3.493

From Table 4.2 it was observed that 't' values for difference between grand score and perfect score in respect of strategy A and strategy B were insignificant ($p < .01$, *d.f.*=24). This indicated that grand scores for strategy A and strategy B were not significantly different from theoretically perfect scores for these strategies. Or, in other words, the teacher adequately adhered to these strategies during experimental teaching.

't' value for control strategy was, however, found to be significant ($p < .01$, *d.f.*=24). This indicated that treatment given to control group was not free from some characteristics of either of the two experimental strategies. To ascertain the extent of this contamination grand score of control strategy was compared with that of Ausubel strategy and Bruner strategy. Table 4.3 shows 't' values for differences between the grand scores of the three strategies.

From Table 4.3 it was observed that difference between the grand scores of strategy A and strategy C as well as that between strategy B and strategy C were highly significant ($p < .01$, *d.f.*=24). This indicated that control treatment was significantly different from the two experimental treatments. Hence, contamination, if any, was negligible.

TABLE 4.3

Significance of Differences Between the Grand Scores of the Three Strategies

Strategies Compared	M_1	σ_1	M_2	σ_2	D	σ_D	t
A—C	4.628	.699	2.172	.568	2.456	.179	13.721
B—C	1.364	.350	2.172	.568	.808	.133	6.076
A—B	4.628	.699	1.364	.350	3.264	.156	20.923

Significant difference between the grand scores of strategy A and strategy B showed that the two experimental treatments in the present study were sufficiently pulled apart. Kertinger (1973, p. 308) asserted that to maximise experimental variance the researcher must "pull" the methods apart as much as possible to make them as unlike as possible.

4.2.0. **Identification of Cognitive Style Sub-Groups**

Conceptual Style Preferences of students in the three groups were identified by their performance on 'Cognitive Style Test' (CST). Students in each group were classified according to their conceptual style preferences. Table 4.4 shows the number of students in stylewise subgroups of each group.

TABLE 4.4

Cognitive Style-Wise Classification of Students in Groups A, B, C

Group	Ausubel A	Bruner B	Control C
Categorical c	18	13	16
Descriptive d	05	06	06
Relational r	14	17	16
Whole w	37	36	38

4.3.0. Descriptive Analysis of the Scores on Intelligence Test (RPM)

Means and Standard Deviations of the scores on intelligence test (RPM) were computed for each group and its style-wise subgroups. Table 4.5 shows these descriptive statistics.

TABLE 4.5

Means and Standard Deviations of the Three Groups and their Style-Wise Subgroups on Intelligence Test (RPM)

Group		A	B	C
W	M	44.4054	42.8611	46.3421
	σ	6.6686	6.6855	6.1217
c	M	45.1667	41.9231	48.5000
	σ	7.4063	8.0464	5.8310
d	M	44.2000	46.1667	46.6667
	σ	4.3244	5.1155	4.8855
r	M	43.5000	42.4119	44.0625
	σ	6.6303	6.0007	6.3190

4.4.0. Descriptive Analysis of the Scores on Previous Knowledge Test (PKT)

Mean and Standard Deviations of the scores on Previous Knowledge Test (PKT) were computed for each group and its style-wise subgroups. Table 4.6 shows these descriptive statistics.

TABLE 4.6

Means and Standard Deviations of the Three Groups and Their Style-Wise Subgroups on Previous Knowledge Test (PKT)

Group		A	B	C
W	M	5.5135	4.1111	4.7895
	σ	1.5023	1.5265	1.5624
c	M	5.9444	4.0000	4.9375
	σ	1.4742	1,3540	1,7308
d	M	5.4000	3.6667	4.8333
	σ	1.1402	1.7512	1.3292
r	M	5.0000	4.3529	4.6250
	σ	1.5134	1.5297	1.4967

4.5.0. Descriptive Analysis of the Scores on Concept Knowledge Test (CKT)

Means and Standard Deviations of the scores on Concept Knowledge Test (CKT) were computed for each group and its style-wise subgroups. Table 4.7 shows these descriptive statistics.

4.6.0. Inferential Analysis of the Scores on Concept Knowledge Test for the Whole Groups A, B, C

This analysis was done to test the major hypotheses I.1., II.1., III.1. For the sake of convenience these hypotheses have been restated.

I.1. There is no significance difference between the mean performance of A and C on CKT.

TABLE 4.7

Means and Standard Deviations of the Three Groups and Their Style-Wise Subgroups on Concept Knowledge Test (CKT)

Group		A	B	C
W	M	7.6757	6.9167	4.7632
	σ	1.6676	1.9328	1.4225
c	M	8.1667	6.9231	4.0000
	σ	1.4653	2.3616	1.4142
d	M	7.2000	6.0000	5.3333
	σ	2.2804	2.0000	1.3663
r	M	7.2143	7.2353	5.3125
	σ	1.6257	1.5219	1.1383

II.1. There is no significant difference between the mean performance of B and C on CKT.

III.1. There is no significant difference between the mean performance of A and B on CKT.

Since the procedure adopted for inferential analysis was "Analysis of Covariance", Bartlett's test was applied to the data on 'Concept Knowledge Test' for the groups A, B, C in order to check the assumption of 'variance homogeneity.' The computations yielded $\chi^2=3.441<\chi^2(2)=5.991$ ($p=.05$). Hence, the assumption of 'variance homogeneity' was fulfilled.

'Analysis of Covariance' was carried out for the dependent variable "Concept Knowledge" taking 'Previous Knowledge' and 'Intelligence' as covariates.

F-value for the treatment variable was found to be 27.358 (D.F.=2,106) which was significant ($p < .01$) and indicated that there were significant mean differences between the performance of the three groups on concept knowledge test. F-values for the two covariates, intelligence and previous knowledge, were insignificant, indicating thereby that the groups did not differ significantly on pre-treatment measures. However, to cancell out even the small differences on these measures, means of dependent variable were adjusted using regression coefficients. The adjusted means for the three groups A, B, C were found to be 7.5894, 6.9812 and 4.7860 respectively.

In order to find the groups which differed significantly with regard to mean performance on Concept Knowledge Test, t-test was employed for testing the significance of difference between two adjusted means. Table 4.8 presents t-values for three pairs of adjusted group means compared, along with their significance.

TABLE 4.8

Significance of Difference Between Adjusted Means of Groups A, B, C on Concept Knowledge Test

Group Means Compared	t-value	D.F.	Level of Significance
A—C	6.909	106	$p < .01$
B—C	5.372	106	$p < .01$
A—B	1.479	106	n.s.

Significant ($p < .01$) t-value for mean-difference between groups A and C indicated that these two groups differed significantly with regard to mean performance on Concept Knowledge Test. Hypothesis I.1. was, therefore, rejected. Moreover, since the difference was in favour of group A, it could be inferred that Ausubel group performed significantly better than control group on Concept Knowledge Test.

Significant ($p < .01$) t-value for mean difference between groups B and C indicated that these two groups differed significantly with regard to mean performance of Concept Knowledge Test. Hypothesis II.1. was, therefore, rejected. Moreover, since the difference was in favour of group B, it could be inferred that Bruner group performed significantly better than control group Concept Knowledge Test.

t-value for mean difference between groups A and B was not significant even at .05 level. This indicated that these two groups did not differ significantly with regard to mean performance on Concept Knowledge Test. Hypotheses III.1. was, therefore, accepted. Although mean-difference was in favour of group A, it could not be inferred that group A performed better than group B. The performance of Ausubel group and Bruner group on Concept Knowledge Test could be treated as equivalent.

4.6.1. Inferential Analysis of the Scores on Concept Knowledge Test for Categorical Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.c.*1. There is no significant difference between the mean performance of Ac and Cc on CKT.

*ii.c.*1. There is no significant difference between the mean performance of Bc and Cc on CKT.

*iii.c.*1. There is no significant difference between the mean performance of Ac and Bc on CKT.

Homogeneity of variance was first checked by applying Bartlett's Test to the data on Concept Knowledge Test for the subgroups Ac, Bc, Cc. Computations yielded $\chi^2=4.5271<\chi^2(2)=5.991$ ($p=.05$) indicating variance homogeneity.

Analysis of Covariance was carried out for the dependent variable "Concept Knowledge" taking 'Intelligence' and 'Previous Knowledge' as covariates.

Significant ($p<.01$) F-value 21.281 (D.F. : 2, 42) for treatment variable indicated that the subgroups differed significantly with regard to mean performance on Concept Knowledge Test.

Although F-values for the two covariates were insignificant, the means of dependent variable were adjusted to cancel out pre-treatment differences, howsoever small, using regression coefficients. The adjusted means for the three subgroups Ac, Bc, Cc were found to be 8.0865, 7.0684 and 3.9722 respectively.

To identify subgroups which differed significantly with regard to mean performance on Concept Knowledge Test, t-test was employed for testing the significance of differences between two adjusted means. Table 4.9 presents t-values for three pairs of adjusted subgroup means compared and their significance.

TABLE 4.9

Significance of Difference Between Adjusted Means of Subgroups Ac, Bc, Cc on Concept Knowledge Test

Subgroup Means Compared	t-value	D.F.	Level of Significance
Ac—Cc	6.263	42	$p < .01$
Bc—Cc	4.337	42	$p < .01$
Ac—Bc	1.463	42	n.s.

Since t-value for mean difference of subgroups Ac and Cc on Concept Knowledge Test was significant ($p < .01$), hypothesis *i.c.*1. was rejected. The difference was in favour of Ac. Hence, categorical style subgroup of Ausubel group performed significantly better than its counterpart in control group in Concept Knowledge Test.

Since t-value for mean difference of subgroups Bc and Cc on Concept Knowledge Test was significant ($p < .01$), hypothesis *ii.c.*1. was rejected. The difference was in favour of Bc. Hence, categorical style subgroup of Bruner group performed significantly better than its counterpart in control group on Concept Knowledge Test.

t-value for mean difference on Concept Knowledge Test for the subgroups Ac and Bc was not significant. Hence, hypothesis *iii.c.*1. was accepted. Categorical style subgroups of Ausubel and Bruner groups performed equally on Concept Knowledge Test.

4.6.2. Inferential Analysis of the Scores on Concept Knowledge Test for Descriptive Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.d.*1. There is no significant difference between the mean performance of Ad and Cd on CKT.

*ii.d.*1. There is no significant difference between the mean performance of Bd and Cd on CKT.

*iii.d.*1. There is no significant difference between the mean performance of Ad and Bd on CKT.

Bartlett's Test was first applied to check the assumption of 'homogeneity of variance' for the scores of subgroups Ad, Bd, Cd on Concept Knowledge Test. $\chi^2=.1059<\chi^2(2)=5.991$ $(p=.05)$ indicated variance homogeneity.

Analysis of Covariance was, therefore, carried out for the dependent variable "Concept Knowledge" taking 'Intelligence' and 'Previous Knowledge' as covariates.

Since F-value 1.454 (D.F.=2,12) for the treatment variable was insignificant even at .05 level of significance, mean performance of the subgroups Ad, Bd, Cd on Concept Knowledge Test did not differ significantly. Hypotheses *i.d.*1., *ii.d.*1., *iii.d.*1. were, therefore, accepted. The performance of descriptive style subgroups of Ausubel and Bruner groups on Concept Knowledge Test could be treated as same as that of their counterpart in control group.

4.6.3. Inferential Analysis of the Scores on Concept Knowledge Test for Relational Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.r.*1. There is no significant difference between the mean performance of Ar and Cr on CKT.

*ii.r.*1. There is no significant difference between the mean performance of Br and Cr on CKT.

*iii.r.*1. There is no significant difference between the mean performance of Ar and Br on CKT.

Bartlett's test was first applied to the data of subgroups Ar, Br, Cr on Concept Knowledge Test in order to check the assumption of 'homogeneity of variance'. $\chi^2=1.8924<\chi^2(2)=5.991$ ($p=.05$) fulfilled the assumption.

Analysis of covariance was carried out for the dependent variable "Concept Knowledge" taking 'Intelligence' and 'Previous Knowledge' as covariates.

Significant ($p<.01$) F-value 8.786 (D.F. : 2, 42) for treatment variable indicated that the subgroups differed significantly with regard to mean performance on Concept Knowledge Test. Although F-values for the two covariates were insignificant, the means of dependent variable were adjusted to cancel out pre-treatment differences, howsoever small, using regression coefficients. The adjusted means for the three subgroups Ar, Br, Cr were found to be 7.1657, 7.2512 and 5.3381 respectively.

To identify subgroups which differed significantly with regard to mean performance on Concept Knowledge Test, t-test was employed for testing the significance of difference between two adjusted means. Table 4.10 presents t-values for

TABLE 4.10

Significance of Difference Between Adjusted Means of Subgroups Ar, Br, Cr on Concept Knowledge Test

Subgroup Means Compared	t-value	D.F.	Level of Significance
Ar—Cr	3.419	42	$p<.01$
Br—Cr	3.760	42	$p<.01$
Ar—Br	0.162	42	n.s.

three pairs of adjusted subgroup means compared and their significance.

Since t-value for mean difference of subgroups Ar and Cr on Concept Knowledge Test was significant ($p < .01$), hypothesis *i.r.*1. was rejected. The difference was in favour of Ar. Hence, Relational Style subgroup of Ausubel group performed significantly better than its counterpart in control group on Concept Knowledge Test.

Since t-value for mean difference of subgroups Br and Cr on Concept Knowledge Test was significant ($p < .01$), hypothesis *ii.r.*1. was rejected. The difference was in favour of Br. Hence, Relational Style subgroup of Bruner group performed significantly better than its counterpart in control group on Concept Knowledge Test.

t-value for mean difference on Concept Knowledge Test for the subgroups Ar and Br was not significant. Hence, hypothesis *iii.r.*1. was accepted. Relational Style subgroups of Ausubel and Bruner groups performed equally on Concept Knowledge Test.

4.7.0. Descriptive Analysis of the Scores on Concept Transfer Test (CTT)

Means and Standard Deviations of the scores on Concept Transfer Test (CTT) were computed for each group and its style-wise subgroups. Table 4.11 shows these descriptive statistics.

4.8.0. Inferential Analysis of the Scores on Concept Transfer Test for the Whole Groups A, B, C

This analysis was done to test the major hypotheses I.2., II.2., III.2. For the sake of convenience these hypotheses have been restated.

I.2. There is no significant difference between the mean performance of A and C on CTT.

II.2. There is no significant difference between the mean performance of B and C on CTT.

III.2. There is no significant difference between the mean performance of A and B on CTT.

TABLE 4.11

Means and Standard Deviations of the Three Groups and Their Style-Wise Subgroups on Concept Transfer Test (CTT)

Group		A	B	C
W	M	6 2973	5.2500	4.3421
	σ	1.8836	1.4015	1.1689
c	M	6.8889	5.2308	4.0625
	σ	1.8752	1.4806	1.2894
d	M	6.6000	4.5000	4.6667
	σ	1.6733	1.3784	0.8165
r	M	5.4286	5.5294	4.5000
	σ	1.7415	1.4902	1.3102

Since the statistical technique adopted for inferential analysis was "Analysis of Covariance", it was necessary to check whether an assumption of 'homogeneity of variance' was fulfilled. Bartlett's test was, therefore, applied to the data on Concept Transfer Test for the groups A, B, C. Computations yielded $\chi^2=8.4901$ (D.F.=2).

Since, $x^2=8.4901>\chi^2(2)=5.991$ ($p=.05$) null hypothesis of equal variances could not be accepted at .05 level of significance. However, $\chi^2=8.4901<\chi^2(2)=9.210$ ($p=.01$). Hence, the assumption of homogeneity of variance could be accepted at .01 level of significance, although it involved the risk of committing Type II error.

The Analysis of Covariance was, nevertheless, carried out for the dependent variable "Concept Transfer" taking 'Intelligence' and 'Concept Knowledge' as covariates.

F-value 3.582 (D.F. : 2, 106) for the treatment variable was significant ($p<.05$). Hence, significant mean differences between the performance of the three groups on concept transfer test were indicated. Since F-value 19.693 (D.F. : 1,106) for the covariates 2 (Concept Knowledge) was significant ($p < .01$), differences on this measure, and also on intelligence (though meagre), were cancelled out by computing adjusted means for the dependent variable, using regression coefficients. The adjusted means for the three groups A, B, C were found to be 5.8574, 5.0994 and 4.9131 respectively.

In order to find the groups which differed significantly with regard to mean performance on Concept Transfer Test, t-test was employed for testing the significance of difference between two adjusted means. Table 4.12 presents t-values for three pairs of adjusted group-means compared, along with their significance.

TABLE 4.12

Significance of Difference Between Adjusted Means of Groups A, B, C on Concept Transfer Test

Group Means Compared	t-value	D.F.	Level of Significance
A—C	2.671	106	$p < .01$
B—C	0.523	106	n.s
A—B	2.116	106	$p < .05$

Significant ($p < .01$) t-value for mean difference between groups A and C indicated that these two groups differed significantly with regard to mean performance on concept transfer test. Hypothesis I.2. was, therefore, rejected. Moreover, since the difference was in favour of group A, it could be inferred that Ausubel group performed significantly better than control group on Concept Transfer Test.

t-value for mean-difference between groups B and C was not significant. This indicated that these two groups did not differ significantly with regard to mean performance on Concept Transfer Test. Hypothesis II.2. was, therefore, accepted. Although mean-difference was in favour of group B, it could not be inferred that group B performed better than group C. The performance of Bruner group on Concept Transfer Test could be treated as at par with that of control group.

Significant ($p<.05$) t-value for mean-difference between groups A and B indicated that there was significant difference between the mean performance of these two groups on Concept Transfer Fest. Hypothesis III.2. was, therefore, rejected. Moreover, since the difference was in favour of group A, it could be inferred that Ausubel group performed significantly better than Bruner group on Concept Transfer Test.

4.8.1. Inferential Analysis of the Scores on Concept Transfer Test for Categorical Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.c.*2. There is no significant difference between the mean performance of Ac and Cc on CTT.

*ii.c.*2. There is no significant difference between the mean performance of Bc and Cc on CTT.

*iii.c.*2. There is no significant difference between the mean performance of Ac and Bc on CTT.

Homogeneity of variance was first checked by applying Bartlett's test to the data of subgroups Ac, Bc, Cc on Concept Transfer Test. Computations yielded $\chi^2=2.242$ (D.F.=2).

Since, $\chi^2=2.242<\chi^2(2)=5.991$ ($p=.05$) null hypothesis of equal variance was accepted. An assumption of 'variance homogeneity' was, thus, fulfilled for the scores of subgroups Ac, Bc, Cc on Concept Transfer Test.

Analysis of covariance was then carried out for the dependent variable "Concept Transfer" taking 'Intelligence' and 'Previous Knowledge' as covariates.

Insignificant F-value 2.538 (D.F.=2, 42) for the treatment variable indicated that the subgroups Ac, Bc, Cc did not differ significantly with regard to their mean performance on concept

Transfer Test. Hypotheses *i.c.*2., *ii.c.*2., and *iii.c.*2. were, therefore, accepted. It could be inferred that categorical style subgroups of Ausubel and Bruner groups were equivalent to control group and also equivalent among themselves in respect of mean-performance on Concept Transfer Test.

4.8.2. Inferential Analysis of the Scores on Concept Transfer Test for Descriptive Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.d.*2. There is no significant difference between the mean performance of Ad and Cd on CTT.

*ii.d.*2. There is no significant difference between the mean performance of Bd and Cd on CTT.

*iii.d.*2. There is no significant difference between the mean performance of Ad and Bd on CTT.

Bartlett's test was applied to the data of subgroups Ad, Bd, Cd on Concept Transfer Test to check the assumption of 'homogeneity of variance'. Computations yielded $\chi^2=2.7043$.

Since, $\chi^2=2.7043<\chi^2(2)=5.991$ ($p=.95$) null hypothesis of equal variance was accepted and the assumption of 'variance homogeneity' fulfilled.

Analysis of covariance was then carried out for the dependent variable "Concept Transfer" taking 'Intelligence' and 'Previous Knowledge' as covariates.

Insignificant F-value 2.267 (D.F.=2, 12) for the treatment variable indicated that the subgroups Ad, Bd, Cd did not differ significantly with regard to their mean performance on Concept Transfer Test. Hypotheses *i.d.*2., *ii.d.*2., *iii.d.*2. were, therefore, accepted. It could be inferred that Ausubel, Bruner and control groups were equivalent in respect of mean performance on Concept Transfer Test.

4.8.3. Inferential Analysis of the Scores on Concept Transfer Test for Relational Style Subgroups of Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.r.*2. There is no significant difference between the mean performance of Ar and Cr on CTT.

*ii.r.*2. There is no significant difference between the mean performance of Br and Cr on CTT.

*iii.r.*2. There is no significant difference between the mean performance of Ar and Br on CTT.

Bartlett's test was applied to the data of subgroups Ar, Br, Cr on Concept Transfer Test to check the assumption of 'homogeneity of variance'. Computations yielded $\chi^2=2.4346<\chi^2(2)=5.991$ ($p=.05$) indicating variance homogeneity.

Analysis of covariance was carried out for the dependent variable "Concept Transfer" taking 'Intelligence' and 'Concept Knowledge' as covariates.

Insignificant F-value 0.428 (D.F.=2, 42) for the treatment variable indicated that the subgroups Ar, Br, Cr did not differ significantly with regard to their mean performance on Concept Transfer Test. Hypotheses *i.r.*2., *ii.r.*2., *iii.r.*2. were, therefore, accepted. It could be inferred that Ausubel, Bruner and control groups were equivalent in respect of mean performance on Concept Transfer Test.

4.9.0. Descriptive Analysis of the Scores on Heuristic Transfer Test (HTT)

Means and standard deviations of the scores on Heuristic Transfer Test (HTT) were computed for each group and its style-wise subgroups. Table 4.13 shows these descriptive statistics.

4.10.0. Inferential Analysis of the Scores on Heuristic Transfer Test for the Whole Groups A, B, C

This analysis was done to test the major hypotheses I.3., II.3., III.3. For the sake of convenience these hypotheses have been restated.

I.3. There is no significant difference between the mean performance of A and C on HTT.

II.3. There is no significant difference between the mean performance of B and C on HTT.

TABLE 4.13

Means and Standard Deviations of the Three Groups and Their Style-Wise Subgroups on Heuristic Transfer Test

Group		A	B	C
W	M	3.2703	6.1111	1.1579
	σ	1.7102	1.6695	1.2846
c	M	3.8333	5.6923	0.8125
	σ	1.7905	1.5484	1.1087
d	M	2.4000	5.5000	1.1667
	σ	1.6733	2.0736	0.4082
r	M	2.8571	6.6471	1.5000
	σ	1.4601	1.5387	1.5916

III.3. There is no significant difference between the mean performance of A and B on HTT.

Since the statistical technique adopted for inferential analysis was "Analysis of Covariance", it was necessary to check whether an assumption of 'homogeneity of variance' was fulfilled. Bartlett's test was, therefore, applied to the data on Heuristic Transfer Test for the groups A, B, C. Computations yielded $\chi^2=3.384<\chi^2(2)=5.991$ $(p=.05)$ indicating homogeneity of variance.

Analysis of covariance was carried out for the dependent variable "Heuristic Transfer" taking 'Intelligence' and 'Concept Knowledge' as covariates.

Significant $(p<.01)$ F-value 98.121 (D.F.=2, 106) for the treatment variable indicated that the groups differed significantly with regard to mean performance on Heuristic Transfer Test. Since F-values (13.606 and 44.668) for both the covariates were also significant $(p<.01)$, it was necessary to cancel out differences on these measures. This was done by adjusting the means of dependent variable using regression coefficients for the two covariates. The adjusted means for the three groups A, B, C were found to be 2.6774, 5.9944 and 1.8457 respectively.

In order to find the groups which differed significantly with regard to mean performance of Heuristic Transfer Test, *t*-test was employed for testing the significance of difference between two adjusted means. Table 4.14 presents *t*-values for three pairs of adjusted group means compared and their significance.

TABLE 4.14

Significance of Difference Between Adjusted Means of Groups A, B, C on Heuristic Transfer Test

Group Means Compared	t-value	D.F.	Level of Significance
A—C	2.592	106	$p < .05$
B—C	12.841	106	$p < .01$
A—B	10.200	106	$p < .01$

t-value for mean difference between groups A and C was significant ($p < .05$) which indicated that these two groups differed significantly with regard to mean performance on Heuristic Transfer Test. Hypothesis I.3. was, therefore, rejected at .05 level of significance. The difference was in favour of group A. Hence, it could be inferred that Ausubel group performed better than control group on Heuristic Transfer Test.

Significant ($p < .01$) *t*-value for mean difference between groups B and C indicated that these two groups differed significantly with regard to mean performance on Heuristic Transfer Test. Hypothesis II.3. was, therefore, rejected. Moreover, since the difference was in favour of group B, it could be inferred that Bruner group performed significantly better than control group on Heuristic Transfer Test.

t-value for mean difference between groups A and B was also significant ($p < .01$), indicating thereby that there was significant difference between the mean performance of these two groups on Heuristic Transfer Test. Hypothesis III.3. was, therefore, rejected. The difference was in favour of group B. Hence, it could be inferred that Bruner group performed significantly better than Ausubel group on Heuristic Transfer Test.

4.10.1. Inferential Analysis of the Scores on Heuristic Transfer Test for Categorial Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.c.*3. There is no significant difference between the mean performance of Ac and Cc on HTT.

*ii.c.*3. There is no significant difference between the mean performance of Bc and Cc on HTT.

*iii.c.*3. There is no significant difference between the mean performance of Ac and Bc on HTT.

Homogeneity of variance was first checked by applying Bartlett's test to the data of subgroups Ac, Bc, Cc on Heuristic Transfer Test. $\chi^2 = 3.395 < \chi^2(2) = 5.991$ ($p = .05$) indicated variance homogeneity.

Analysis of covariance was then carried out for the dependent variable "Heuristic Transfer" taking 'Intelligence' and 'Concept Knowledge' as covariates.

Significant ($p < .01$) F-value 26.129 (D.F.=2, 42) for the treatment variable indicated that the subgroups differed significantly with regard to mean performance on Heuristic Transfer Test. Since F-values (4.117 and 11.983) for the two covariates were also significant, differences on these measures were cancelled out by computing adjusted means of the dependent variable using regression coefficients. The adjusted means for the three subgroups Ac, Bc, Cc were found to be 3.1339, 5.7014 and 1.5920 respectively.

To identify subgroups which differed significantly with regard to mean performance on Heuristic Transfer Test, *t*-test was employed for testing the significance of difference between two adjusted means. Table 4.15 presents *t*-values for three pairs of adjusted sub-group-means compared and their significance.

TABLE 4.15

Significance of Difference Between Adjusted Means of Subgroups Ac, Bc, Cc on Heuristic Transfer Test

Subgroups Means Compared	t-value	D.F.	Level of Significance
Ac—Cc	2.990	42	$p < .01$
Bc—Cc	7.333	42	$p < .01$
Ac—Bc	4.700	42	$p < .01$

Since *t*-value for mean difference of subgroups Ac and Cc on Heuristic Transfer Test was significant ($p < .01$), hypothesis *i.c.*3. was rejected. The difference was in favour of Ac. Hence, categorical style subgroup of Ausubel group performed significantly better than its counterpart in control group on Heuristic Transfer Test.

Since *t*-value for mean difference of subgroups Bc and Cc on Heuristic Transfer Test was significant ($p < .01$), hypothesis *ii.c.*3. was rejected. The difference was in favour of Bc. Hence, categorical style subgroup of Bruner group performed significantly better than its counterpart in control group on heuristic transfer test.

t-value for mean difference between the subgroups Ac and Bc on Heuristic Transfer Test was also significant ($p < .01$). Hypothesis *iii.c.*3. was, therefore, rejected. Mean difference was in favour of Bc. Hence, it could be inferred that categorical style subgroups of Bruner group performed significantly better than categorical style subgroup of Ausubel group on Heuristic Transfer Test.

4.10.2. Inferential Analysis of the Scores on Heuristic Transfer Test for Descriptive Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.d.*3. There is no significant difference between the mean performance of Ad and Cd on HTT.

*ii.d.*3. There is no significant difference between the mean performance of Bd and Cd on HTT.

*iii.d.*3. There is no significant difference between the mean performance of Ad and Bd on HTT.

Bartlett's test was applied to the data of subgroups Ad, Bd, Cd on Heuristic Transfer Test to check the assumption of 'homogeneity of variance'. Computations yielded $\chi^2=8.935$ (D.F.$=2$).

Since, $\chi^2=8.932<\chi^2(2)=9.210$ ($p=.01$), null hypothesis of equal variance could be accepted at .01 level of significance. The assumption of 'homogeneity of variance' was thus fulfilled ($p=.01$). This, however, involved the risk of committing Type II error.

Nevertheless, analysis of covariance was carried out for the dependent variable "Heuristic Transfer" taking 'Intelligence' and 'Concept Knowledge' as covariates.

Significant ($p<.01$) F-value 16.794 (D.F.$=2,12$) for the treatment variable indicated that the subgroups differed significantly with regard to mean performance on Heuristic Transfer Test. F-value for covariate 1 (Intelligence) was insignificant but that for covariate 2 (concept knowledge) was 7.295 (D.F.$=12$) significant ($p<.05$). The differences on both of these measures were, however, cancelled out by computing adjusted means for the dependent variable using regression coefficients. Adjusted means for the three subgroups Ad, Bd, Cd were found to be 1.7764, 5.5563 and 1.6301 respectively.

To identify subgroups which differed significantly with regard to mean performance on Heuristic Transfer Test, t-test was employed for testing the significance of difference between two adjusted means. Table 4.16 presents *t*-values for three pairs of adjusted subgroup means compared and their significance.

t-value for mean difference between subgroups Ad and Cd on Heuristic Transfer Test was insignificant. Hence, hypothesis *i.d.*3. was accepted. Performance of descriptive style subgroup

TABLE 4.16

Significance of Difference Between Adjusted Means of Subgroups Ad, Bd, Cd on Heuristic Transfer Test

Subgroup Means Compared	t-value	D.F.	Level of Significance
Ad—Cd	0.170	12	n.s.
Bd—Cd	4.791	12	$p<.01$
Ad—Bd	4.397	12	$p<.01$

of Ausubel group could be treated as equivalent to that of its counterpart in control group on Heuristic Transfer Test.

Since t-value for mean difference of subgroups Bd and Cd on Heuristic Transfer Test was significant, hypothesis *ii.d.*3. was rejected. The difference was in favour of group Bd. Hence, descriptive style subgroup of Bruner group performed significantly better than its counterpart in control group on Heuristic Transfer Test.

Significant ($p<.01$) *t*-value for mean difference of subgroups Ad and Bd indicated that these two subgroups differed significantly with regard to mean performance on Heuristic Transfer Test. Hypothesis *iii.d.*3. was, therefore, rejected. The difference being in favour of subgroup Bd it could be inferred that descriptive style subgroup of Bruner group performed significantly better than descriptive style subgroup of Ausubel group on Heuristic Transfer Test.

4.10.3. Inferential Analysis of the Scores on Heuristic Transfer Test for Relational Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

i.r.3. There is no significant difference between the mean performance of Ar and Cr on HTT.

ii.r.3. There is no significant difference between the mean performance of Br and Cr on HTT.

iii.r.3. There is no significant difference between the mean performance of Ar and Br on HTT.

Homogeneity of variance was first checked by applying Bartlett's test to the data of subgroups Ar, Br, Cr on Heuristic Transfer Test. Computations yielded $\chi^2 = .0974$.

Since, $\chi^2 = .0974 < \chi^2(2) = 5.991$ $(p = .05)$ assumption of 'variance homogeneity' was fulfilled for the scores of subgroups Ar, Br, Cr on Heuristic Transfer Test.

Analysis of covariance was carried out for the dependent variable "Heuristic Transfer" taking 'Intelligence' and 'Concept Knowledge' as covariates.

Significant $(p < .01)$ F-value 55.896 (D.F.=2, 42) for the treatment variable indicated that the subgroups differed significantly with regard to mean performance on Heuristic Transfer Test. F-values for the two covariates were 12.557 and 15.783 (D.F. : 1, 42) significant $(p < .01)$. Hence, the subgroups differed significantly on these measures. The means of dependent variable were, therefore, adjusted to cancel out these differences. Regression coefficients for the two covariates were used for the purpose. Adjusted means of the subgroups Ar, Br, Cr on Heuristic Transfer Test were computed as 2.5060, 6.3992 and 2.0705 respectively.

In order to find out the subgroups which differed significantly in mean performance on Heuristic Transfer Test, *t*-test was employed for testing the significance of difference between two adjusted means. Table 4.17 presents *t*-values for three pairs of adjusted subgroup means compared and their significance.

Since *t*-value for mean difference of subgroups Ar and Cr on Heuristic Transfer Test was insignificant, the subgroups did not differ significantly with regard to mean performance on this test. Hypothesis *i.r*.3. was, therefore, accepted. It could be

TABLE 4.17

Significance of Difference Between Adjusted Means of Subgroups Ar, Br, Cr on Heuristic Transfer Test

Subgroup Means Compared	t-value	D.F.	Level of Significance
Ar—Cr	0.899	42	n.s.
Br—Cr	9.392	42	p<.01
Ar—Br	8.152	42	p<.01

inferred that relational style subgroup of Ausubel group performed equally as its counterpart in the control group on Heuristic Transfer Test.

t-value for mean difference of subgroups Br—Cr on Heuristic Tranfer Test was significant ($p<.01$). Hypothesis *ii.r.*3. was, therefore, rejected. The mean difference was in favour of subgroup Br. Hence, it could be inferred that relational style subgroup of Bruner group performed significantly better than its counterpart in control group on Heuristic Transfer Test.

t-value for mean difference between subgroups Ar and Br was also significant ($p<.01$). Hypothesis *iii.r.*3. was, thus, rejected. Since mean difference was in favour of subgroup Br, it could be inferred that relational style subgroup of Bruner group performed significantly better than relational style subgroup of Ausubel group on Heuristic Transfer Test.

4.11.0. Descriptive Analysis of the Scores on Short-Term Retention Test

Means and standard deviations of the scores on short-term retention test were computed for each group and its style-wise subgroups. Table 4.18 shows these descriptive statistics.

4.12.0. Inferential Analysis of the Scores on Short-Term Retention Test for the Whole Groups A, B, C

This analysis was done to test the major hypotheses **I.4.**,

TABLE 4.18

Means and Standard Deviations of the Three Groups and their Style-wise Subgroups on Short-Term Retention Test

Group		A	B	C
W	M	3.3514	6.4444	2.4727
	σ	2.5300	1.4029	1.3302
c	M	2.9444	6.3846	2.2500
	σ	2.4608	1.9807	1.0000
d	M	4.4000	6.1667	2.5000
	σ	2.8810	0.7528	1,8708
r	M	3.5000	6.5882	2.6875
	σ	2.5646	1.0037	1.4705

II.4., III.4. For the sake of convenience these hypotheses have been restated.

I.4. There is no significant difference between the mean performance of A and C on SRT.

II.4. There is no significant difference between the mean performance of B and C on SRT.

III.4. There is no significant difference between the mean performance of A and B on SRT.

Bartlett's test was applied to the data of groups A, B, C on Short-Term Retention Test. Computations yielded $\chi^2=19.552$ (D.F.=2).

Since, $\chi^2=19.552>\chi^2(2)=9.210$ ($p=.01$), null hypothesis of equal variance was untenable. An assumption of 'variance homogeneity' was, thus, unfulfilled, thereby restricting proper interpretation of the results obtained from analysis of covariance.

Nevertheless, analysis of covariance was carried out for the dependent variable "Short-Term Retention" taking 'Intelligence' and 'Concept Knowledge' as covariates.

Significant ($p < .01$) F-value 45.028 (D.F.=2, 106) for the treatment variable indicated that mean performance of the groups differed significantly on Short-Term Retention Test. Although F-values for the two covariates were insignificant, means of dependent variable were adjusted to cancel out differences on covariables, howsoever small, using regression coefficients. Adjusted means of the three groups A, B, C on Short-Term Retention Test were computed as 3.2450, 6.4662 and 2.5566 respectively.

In order to find the groups which differed significantly with regard to mean performance on Short-Term Retention Test, *t*-test was employed for testing the significance of difference between two adjusted means. Table 4.19 presents *t*-values for three pairs of adjusted group means compared and their significance.

TABLE 4.19

Significance of Difference Between Adjusted Means of Groups A, B, C on Short-Term Retention Test

Group Means Compared	t-value	D.F.	Level of Significance
A—C	1.489	106	n.s.
B—C	8.396	106	p<.01
A—B	6.872	106	p<.01

t-value for mean difference between groups A and C was insignificant. Hence, the two groups did not differ significantly with regard to mean performance on Short-Term Retention Test. Hypothesis I.4. was, therefore, accepted. It could be inferred that the performance of Ausubel group was equivalent to that of control group on Short-Term Retention Test.

Significant ($p-.01$) t-value for mean difference between groups B and C indicated that these two groups differed significantly with regard to mean performance on Short-Term Retention Test. Hypothesis II.4. was, therefore, rejected. Since the difference was in favour of group B, it could be inferred that Bruner group performed significantly better than control group on Short-Term Retention Test.

t-value for mean difference between groups A and B was also significant ($p<.01$), indicating thereby that there was significant mean difference between the performance of these two groups on Short-Term Retention Test. Hypothesis III.4. was, therefore, rejected. The difference was in favour of group B. Hence, it could be inferred that Bruner group performed significantly better than Ausubel group on Short-Term Retention Test.

4.12.1. Inferential Analysis of the Scores on Short-Term Retention Test for Categorical Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.c.*4. There is no significant difference between the mean performance of Ac and Cc on SRT.

*ii.c.*4. There is no significant difference between the mean performance of Bc and Cc on SRT.

*iii.c.*4. There is no significant difference between the mean performance of Ac and Bc on SRT.

Bartlett's test was applied to the data on Short-Term Retention Test for the subgroups Ac, Bc, Cc to check the assumption of 'variance homogeneity'. Computations yielded $\chi^2=17.269$.

Since, $\chi^2=17.269>\chi^2(2)=9.210$ ($p=.01$), the assumption of 'variance homogeneity' was not satisfied. Nevertheless, analysis of covariance was carried out for the dependent variable "Short-Term Retention" taking 'Intelligence' and 'Concept Knowledge' as covariates.

Since F-value 15.373 (D.F.=2, 42) for the treatment variable was significant ($p<.01$), mean performance of the subgroups differed significantly on Short-Term Retention Test.

Although F-values for the two covariates were insignificant, differences, howsoever meagre, on these variables were neutralized by adjusting the means of dependent variable using regression coefficients. Adjusted means of the three subgroups Ac, Bc, Cc on Short-Term Retention Test were computed as 2.8822, 6.4172 and 2.2935 respectively.

In order to find the subgroups which differed significantly with regard to mean performance on Short-Term Retention Test, t-test was employed for testing the significance of difference between two adjusted means. Table 4.20 presents *t*-values for three pairs of subgroup means compared and their significance.

TABLE 4.20

Significance of Difference Between Adjusted Means of Groups Ac, Bc, Cc on Short-Term Retention Test

Subgroup Means Compared	t-value	D.F.	Level of Significance
Ac—Cc	0.757	42	n.s.
Bc—Cc	4.879	42	p.<.01
Ac—Bc	4.290	42	p.<.01

t-value for mean difference between subgroups Ac and Cc on Short-Term Retention Test was insignificant. Hence, the subgroups did not differ significantly with regard to mean performance on Short-Term Retention Test. Hypothesis *i.c.* 4. was, therefore, accepted. It could be inferred that categorical style subgroup of Ausubel group performed equally as its counterpart in control group on Short-Term Retention Test.

Since *t*-value for mean difference between subgroups Bc and Cc was significant ($p < .01$), these two groups differed significantly with regard to mean performance on Short-Term Retention Test. Hypothesis *ii.c.*4. was, therefore, rejected. Mean difference was in favour of subgroup Bc. Hence, it could be inferred that categorical style subgroup of Bruner group performed significantly better than its counterpart in control group on Short-Term Retention Test.

Significant ($p<.01$) t-value for mean difference between subgroups Ac and Bc indicated that mean performance of these two subgroups on Short-Term Retention Test differed significantly. Hypothesis *iii.c.*4. was, therefore, rejected. Since the difference was in favour of subgroup Bc, it could be inferred that categorical style subgroup of Bruner group performed significantly better than categorical style subgroup of Ausubel group on Short-Term Retention Test.

4.12.2. Inferential Analysis of the Scores on Short-Term Retention Test for Descriptive Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.d.*4. There is no significant difference between the mean performance of Ad and Cd on SRT.

*ii.d.*4. There is no significant difference between the mean performance of Bd and Cd on SRT.

*iii.d.*4. There is no significant difference between the mean performance of Ad and Bd on SRT.

Bartlett's test applied to the data of subgroups Ad, Bd, Cd on Short-Term Retention Test yielded $\chi^2=6.289<x^2(2)=9.210$ ($p=.01$) indicating variance homogeneity.

Analysis of covariance was carried out for the dependent variable "Short-Term Retention" taking 'Intelligence' and 'Concept Knowledge' as covariates.

F-value 4.860 (D.F.=2, 12) for the treatment variable was significant ($p<.05$). Hence, the subgroups differed significantly with regard to mean performance on Short-Term Retention Test. F-value for the covariates were insignificant. However, to annul even small differences on these variables, the means of dependent variable were adjusted using regression coefficients. Adjusted means of the three groups Ad, Bd, Cd on Short-Term

Retention Test were computed as 4.3307, 6.1993 and 2.5251 respectively.

In order to identify subgroups which differed significantly in respect of mean performance on Short-Term Retention Test, t-test was employed to test the significance of difference between the adjusted means. Table 4.21 presents t-values for the three pairs of subgroups means compared and their significance.

TABLE 4.21

Significance of Difference Between Adjusted Means of Subgroups Ad, Bd, Cd on Short-Term Retention Test

Subgroup Means Compared	t-value	D.F.	Level of Significance
Ad—Cd	1.400	12	n.s.
Bd—Cd	2.998	12	$p<.05$
Ad—Bd	1.449	12	n.s.

Since t-value for mean difference between subgroups Ad and Cd was insignificant, the groups did not differ significantly with regard to mean performance on Short-Term Retention Test. Hypothesis *i.d* 4. was, therefore, accepted. It could be inferred that performance of descriptive style subgroup of Ausubel group was equivalent to that of its counterpart in control group on Short-Term Retention Test.

t-value for mean difference between subgroups Bd and Cd was significant ($p<.05$). The subgroups, therefore, differed significantly with regard to mean performance on Short-Term Retention Test. Hypothesis *ii d*.4. was, thus, rejected. The difference was in favour of subgroup Bd. Hence, it could be inferred that descriptive style subgroup of Bruner group performed significantly better than its counterpart in control group on Short-Term Retention Test.

Since t-value for mean difference between subgroups Ad and Bd was insignificant, the subgroups did not differ significantly with regard to mean performance on Short-Term Retention Test. Hypothesis *iii.d.*4. was, therefore, accepted. It could be inferred that descriptive style subgroups of Ausubel and Bruner groups performed equally on Short-Term Retention Test.

4.12.3. Inferential Analysis of the Scores on Short-Term Retention Test for Relational Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.r.*4. There is no significant difference between the mean performance of Ar and Cr on SRT.

*ii.r.*4. There is no significant difference between the mean performance of Br and Cr on SRT.

*iii.r.*4. There is no significant difference between the mean performance of Ar and Br on SRT.

Applying Bartlett's test to the data of subgroups Ar, Br, Cr on Short-Term Retention Test yielded $\chi^2 = 11.424$ (D.F.=2).

Since, $\chi^2 = 11.424 > \chi^2(2) = 9.210$ ($p = .01$) null hypothosis of equal variance was rejected, that is, an assumption of 'variance homogeneity' was not satisfied.

Nevertheless, analysis of covariance was carried out for the dependent variable "Short-Term Retention" taking 'Intelligence' and 'Concept Knowledge' as covariates.

F-value 2 2.665 (D.F.=2, 42) for the treatment variable was significant ($p < .01$) which indicated that significant mean differences existed between the subgroup with regard to performance on Short-Term Retention Test. F-value 11.440 (D.F.=1, 42) for covariate 1 (intelligence) was significant at .01 level and F-value 5.013 (D.F.=1, 42) for covariate 2 (concept knowledge) was significant at .05 level. Hence, differences on these covariables were neutralized by computing adjusted means for the dependent variable using regression coefficients. Adjusted means of subgroups Ar, Br, Cr on Short-Term Retention Test were computed as 3.2432, 6.4996 and 3.0489 respectively.

In order to find subgroups which differed significantly with regard to mean performance on Short-Term Retention Test, t-test was employed for testing the significance of difference between two adjusted means. Table 4.22 presents t-values for three pairs of adjusted subgroup means compared and their significance.

TABLE 4.22

Significance of Difference Between Adjusted Means of Subgroups Ar, Br, Cr on Short-Term Retention Test

Subgroup Means Compared	t-value	D.F.	Level of Significance
Ar—Cr	0.322	42	n.s.
Br—Cr	5.942	42	p<.01
Ar—Br	5.408	42	p<.01

t-value for mean difference between subgroups Ar and Cr was insignificant. Hence, these two subgroups did not differ significantly with regard to mean performance on Short-Term Retention Test. Hypothesis *i.r.*4. was, therefore, accepted. It could be inferred that relational style subgroup of Ausubel group performed equally as its counterpart in control group on Short-Term Retention Test.

Since t-value for mean difference between subgroups Br and Cr was significant ($p<.01$), these two subgroups differed significantly with regard to mean performance on Short-Term Retention Test. Hypothesis *ii.r.*4. was, thus, rejected. Mean difference was in favour of subgroup Br. Hence, it could be inferred that relational style subgroup of Bruner group performed significantly better than its counterpart in control group on Short-Term Retention Test.

Significant ($p<.01$) t-value for mean difference between subgroups Ar and Br indicated that these two subgroups differed significantly with regard to mean performance on Short-Term Retention Test. Hypothesis *iii.r.*4. was, therefore, rejected. Since mean difference was in favour of subgroup Br, it could be inferred that relational style subgroup of Bruner group performed significantly better than relational style subgroup of Ausubel group on Short-Term Retention Test.

4.13.0. Descriptive Analysis of the Scores on Long-Term Retention Test (LRT)

Means and Standard Deviations of the scores on Long-Term Retention Test (LRT) were computed for each group and its style-wise subgroups. Table 4.23 shows these descriptive statistics.

TABLE 4.23

Means and Standard Deviations of the Three Groups and Their Style-wise Subgroups on Long-Term Retention Test

Group		A	B	C
W	M	3.0270	5.3056	1.8158
	σ	1.3014	2.0538	0.9824
c	M	3.5000	5.6154	1.8125
	σ	1.4246	5.0223	0.8342
d	M	2.6000	3.6667	2.0000
	σ	0.8944	2.2509	1.0954
r	M	2.5714	5.6471	1.7500
	σ	1.0894	2.0238	1.5916

4.14.0 Inferential Analysis of the Scores on Long-Term Retention Test for the Whole Groups A, B, C

This analysis was done to test the major hypotheses I.5., II.5., III.5. For the sake of convenience these hypotheses have been restated.

I.5. There is no significant difference between the mean performance of A and C on LRT.

II.5. There is no significant difference between the mean performance of B and C on LRT.

III.5. There is no significant difference between the mean performance of A and B on LRT.

Applying Bartlett's test to the data of groups A, B, C on Long-Term Retention Test yielded $\chi^2=11.928$ (D.F.$=2$).

Since $\chi^2=11.928<\chi^2(2)=9.210$ ($p=.01$) null hypothesis of equal variance was untenable. An assumption of 'variance homogeneity' was not satisfied. This has put some restriction on the interpretation of results obtained from analysis of covariance.

Nevertheless, analysis of covariance was carried out for the dependent variable "Long-Term Retention" taking 'Intelligence', 'Concept Knowledge' and 'Short-Term Retention' as covariates.

F-value 14.930 (D.F.$=2$, 105) for treatment variable was significant ($p=.01$). Hence, significant differences between mean performance of the groups on Long-Term Retention Test. F-value 5.893 (D.F.$=1$, 105) for the covariate 3 (Short-Term Retention) was significant ($p<.05$) which indicated that the groups differed significantly on this covariable. Adjustment of the means of dependent variable was, therefore, warranted to cancel out differences on short-term retention. Although F-values for covariates 1 and 2 were insignificant, differences on these variables, howsoever meagre, were also annulled while adjusting means for the dependent variable. Regression coefficients for the three covariates were used for the purpose. Adjusted means of the three groups A, B, C on Long-Term Retention Test were computed as 2.9654, 4,8410 and 2.3159 respectively.

In order to find the groups which differed significantly with regard to mean performance on Long-Term Retention Test, t-test was employed for testing significance of difference between two adjusted means. Table 4.24 presents t-values for the three pairs of group means compared and their significance.

Since t-value for mean difference between groups A and C was insignificant, these two groups did not differ significantly with regard to mean performance on Long-Term Retention Test. Hypothesis I.5. was, therefore, accepted. It could be inferred

TABLE 4.24

Significance of Difference Between Adjusted Means of Groups A, B, C on Long-Term Retention Test

Group Means Compared	t-value	D.F.	Level of Significance
A—C	1.654	105	n.s.
B—C	6.386	105	$p<.01$
A—B	4.713	105	$p<.01$

that the performance ot Ausubel group on Long-Term Retention Test was equivalent to that of control group on the same test.

t-value for mean difference between the performance of group B and group C on Long-Term Retention Test was significant ($p<.01$). This indicated that there was significant difference between mean performance of these two groups on Long-Term Retention Test. Hypothesis II.5. was, therefore, rejected. Since mean difference was in favour of group B, it could be inferred that Bruner group performed significantly better than control group on Long-Term Retention Test.

Significant ($p<.01$) t-value for mean difference between groups A and B indicated that these two groups differed significantly in respect of mean performance on Long-Term Retention Test. Hypothesis III.5. was, therefore, rejected. Since mean difference was in favour of group B, it could be inferred that Bruner group performed significantly better than Ausubel group on Long-Term Retention Test.

4.14.1. Inferential Analysis of the Scores on Long-Term Retention Test for Categorical Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.c.*5. There is no significant difference between the mean performance of Ac and Cc on LRT.

ii.c.5. There is no significant difference between the mean performance of Bc and Cc on LRT.

iii.c.5. There is no significant difference between the mean performance of Ac and Bc on LRT.

Applying Bartlett's test to the data of subgroups Ac, Bc, Cc on Long-Term Retention Test yielded $\chi^2=51.046$ (D.F.=2).

Since $\chi^2=51.046>\chi^2(2)=9.210$ an assumption of 'variance homogeneity' ($p=.01$) was not satisfied; thereby restricting interpretation of results from analysis of covariance.

Nevertheless, analysis of covariance was carried out for the dependent variable "Long-Term Retention" taking 'Intelligence', 'Concept Knowledge' and 'Short-Term Retention' as covariates.

Since F-value 10.273 (D.F.=2, 41) for treatment variable was significant ($p<.01$), the subgroups differed significantly with regard to mean performance on Long-Term Retention Test. F-values for covariates 1 and 2 were insignificant but that 7.726 (D.F.=1, 41) for covariate 3 was significant ($p<.01$). In order to cancel out differences on covariable 3 and also those on covariables 1 and 2 (though meagre), means of dependent variable were adjusted using regression coefficients. Adjusted means of the three subgroups Ac, Bc, Cc on Long-Term Retention Test were computed as 3.8019, 4.9484 and 2,0149 respectively.

In order to find the subgroups which differed significantly with regard to mean performance on Long-Term Retention Test, t-test was employed for testing the significance of difference between two adjusted means. Table 4.25 presents t-value for three pairs of subgroup means compared and their significance.

TABLE 4.25

Significance of Difference Between Adjusted Means of Subgroups Ac, Bc, Cc on Long-Term Retention Test

Subgroup Means Compared	t-value	D.F.	Level of Significance
Ac—Cc	3.388	41	p<.01
Bc—Cc	5.117	41	p<.01
Ac—Bc	2.052	41	p<.05

Significant ($p<.01$) t-value for mean difference between subgroups Ac and Cc indicated that the subgroups differed significantly with regard to mean performance on Long-Term Retention Test. Hypothesis *i.c.*5. was, therefore, rejected. Mean difference was in favour of Ac. Hence, categorical style subgroup of Ausubel group performed significantly better than its counterpart in control group on Long-Term Retention Test.

Significant ($p<.01$) t-value for mean difference between subgroups Bc and Cc indicated that mean performance of these two groups was significantly different on Long Term Retention Test. Hypothesis *ii.c.*5. was, therefore, rejected. Mean difference being in favour of subgroups Bc, it could be inferred that categorical style subgroup of Bruner group performed significantly better than its counterpart in control group on Long-Term Retention Test.

t-value for mean difference between subgroups Ac and Bc was significant ($p<{\cdot}05$). Hence, mean performance of these two subgroups was significantly different on Long-Term Retention Test. Hypothesis *iii.c.*5. was, therefore, rejected, Mean difference was in favour of subgroup Bc. Hence, it could be inferred that categorical style subgroup of Bruner group performed significantly better than categorical style subgroup of Ausubel group on Long-Term Retention Test.

4.14.2. Inferential Analysis of the Scores on Long-Term Retention Test for Descriptive Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.d.*5. There is no significant difference between the mean performance of Ad and Cd on LRT.

*ii.d.*5. There is no significant difference between the mean performance of Bd and Cd on LRT.

*iii.d.*5. There is no significant difference between the mean performance of Ad and Bd on LRT.

Applying Bartlett's test to the data of subgroups Ad, Bd, Cd on Long-Term Retention Test yielded $\chi^3=4.114$ (D.F.=2).

Since $\chi^2 = 4.114 < \chi^2(2) = 5.991 (p = .05)$ null hypothesis of equal variance was accepted, that is, an assumption of 'variance homogeneity' was satisfied.

Analysis of covariance was carried out for dependent variable "Long-Term Retention" taking 'Intelligence', 'Concept Knowledge' and 'Short-Term Retention' as covariates.

F-value 0.442 (D.F.=2, 11) for the treatment variable was insignificant, indicating thereby that the subgroups did not differ significantly with regard to mean performance on Long-Term Retention Test. Employment of t-test for testing significance of difference between two adjusted means was, therefore, redundant. Mean performance of any of the three pairs of subgroups could be treated as equivalent. Consequently, hypotheses *i.d.*5., *ii.d.*5., and *iii.d* 5. were accepted.

4.14.3. Inferential Analysis of the Scores of Long-Term Retention Test for Relational Style Subgroups of the Groups A, B, C

This analysis was done to test the following minor hypotheses :

*i.r.*5. There is no significant difference between the mean performance of Ar and Cr on LRT.

*ii.r.*5. There is no significant difference between the mean performance of Br and Cr on LRT.

*iii.r.*5. There is no significant difference between the mean performance of Ar and Cr on LRT.

Bartlett's test applied to the data of subgroups Ar, Br, Cr on Long-Term Retention Test yielded $\chi^2 = 5.2819$ (D.F.=2).

Since, $\chi^2 = 5.2819 < \chi^2(2) = 5.991$ $(p = .05)$ null hypothesis of equal variance was accepted, that is, an assumption of 'variance homogeneity' was satisfied.

Analysis of covariance was, then, carried out for the dependent variable "Long-Term Retention" taking 'Intelligence', 'Concept Knowledge' and 'Short-Term Retention' as covariates.

F-value 11.551 (D.F.=2, 41) for the treatment variable was significant $(p < .01)$. This indicated existence of significant mean differences between the performance of the three groups on Long-Term Retention Test. Although F-values for all the three covariates were insignificant, differences on these variables, howsoever small, were annulled by computing adjusted means for the dependent variable using regression coefficients. Adjusted means for the three groups Ar, Br, Cr were computed as 2.6288, 5.4006 and 1.9616 respectively.

In order to identify subgroups which differed significantly with regard to mean performance on Long-Term Retention Test, t-test was employed for testing the significance of difference between two adjusted means. Table 4.26 presents the t-values for three pairs of adjusted subgroup means compared and their significance.

TABLE 4.26

Significance of Difference between Adjusted Means of Subgroups Ar, Br, Cr on Long-Term Retention Test

Subgroup Means Compared	t-value	D.F.	Level of Significance
Ar—Cr	1.100	41	n.s.
Br—Cr	5.955	41	$p<.01$
Ar—Br	4.632	41	$p<.01$

t-value for mean difference between subgroups Ar and Cr was insignificant, indicating thereby that the subgroups did not differ significantly with regard to mean performance on Long-Term Retention Test. Hypothesis *i.r.*5. was, therefore, accepted. It could be inferred that relational style subgroup of Ausubel group performed equally as its counterpart in control group on Long-Term Retention Test.

Since t-value for mean difference between subgroups Br and Cr was significant ($p<.01$), these two groups differed significantly with regard to mean performance on Long-Term Retention Test. Hypothesis *ii.r.*5. was, therefore, rejected. Mean difference was in favour of subgroup Br. Hence, it could be inferred that relational style subgroup of Bruner group performed significantly better than its counterpart in control group on Long-Term Retention Test.

Significant ($p < .01$) t-value for mean difference between subgroups Ar and Br indicated that these two subgroups differed significantly with regard to mean performance on Long-Term Retention Test. Hypothesis *iii.r.*5. was, therefore, rejected. Mean difference being in favour of subgroup Br, it could be inferred that relational style subgroup of Bruner group performed significantly better than relational style subgroup of Ausubel group on Long-Term Retention Test.

4.15.0.

Panorama of Analysis

In order to have gestalt view of inferential analyses, outcome from each analysis has been tabulated in Table 4.27. For convenience of discussion, hypotheses have been classified objective-wise as well as criterion-wise.

4.16.0

Bird's Eye-view of Results

Results obtained from inferential analyses have been tabulated in Table 4.28. In order to facilitate drawing of conclusion, results have been classified both objective-wise and criterion-wise.

TABLE 4.27

Review of Inferential Analyses

OBJECTIVES	HYPOTHESES				
	Concept Knowledge	Concept Transfer	Heuristic Transfer	Short-Term Retention	Long-Term Retention
1	2	3	4	5	6
I	I.1. **	I.2. **	I.3. *	I.4. —	I.5. —
i	i.c.1. ** i.d.1. — i.r.1. **	i.c.2. — i.d.2. — i.r.2. —	i.c.3. ** i.d.3. — i.r.3. —	i.c.4. — i.d.4. — i.r.4. —	i.c 5. * i.d 5. — i.r.5. —
II	II.1. **	II 2. —	II.3. **	II.4. **	II.5. **
ii	ii.c.1. ** ii d.1. — ii.r.1. **	ii.c.2. — ii.d.2. — ii r.2. —	ii.c.3. ** ii.d.3. ** ii.r.3. **	ii.c 4. ** ii.d.4. * ii.r.4. **	ii.c.5. ** ii.d.5. — ii r.5. **

(Contd.)

1	2	3	4	5	6
III	III.1. —	III.2. *	III.3. **	III.4. **	III.5. **
iii	iii.c.1. — iii.d.1. — iii.r.1. —	iii.c.2. — iii.d.2. — iii.r.2. —	iii.c.3. ** iii.d.3. ** iii.r.3. **	iii.c.4. ** iii.d.4. — iii.r.4. **	iii.c.5. * iii.d.5. — iii.r 5. **

** rejected ($p<.01$)

* rejected ($p<.05$)

—accepted.

TABLE 4.28

Summary of Results

Objective			CRITERION				
			Concept Knowledge	Concept Transfer	Heuristic Transfer	Short-Term Retention	Long-Term Retention
1	2	3	4	5	6	7	8
Principal	Whole Groups	I	A>C	A>C	A>C	A~C	A~C
		II	B>C	B~C	B>C	B>C	B>C
		III	A~B	A>B	B>A	B>A	B>A
Subsidiary	Categorical Style Subgroups	i	Ac>Cc	Ac~Cc	Ac>Cc	Ac~Cc	Ac>Cc
		ii	Bc>Cc	Bc~Cc	Bc>Cc	Bc>Cc	Bc>Cc
		iii	Ac~Bc	Ac~Bc	Bc>Ac	Bc>Ac	Bc>Ac

(*Contd.*)

1	2	3	4	5	6	7	8
	Descriptive Style Subgroups	i	Ad~Cd	Ad~Cd	Ad~Cd	Ad~Cd	Ad~Cd
		ii	Bd~Cd	Bd~Cd	Bd>Cd	Bd>Cd	Bd~Cd
		iii	Ad~Bd	Ad~Bd	Bd>Ad	Bd~Ad	Bd~Ad
	Relational Style Subgroups	i	Ar>Cr	Ar~Cr	Ar~Cr	Ar~Cr	Ar~Cr
		ii	Br>Cr	Br~Cr	Br>Cr	Br>Cr	Br>Cr
		iii	Ar~Br	Ar~Br	Br>Ar	Br>Ar	Br>Ar

'>' significantly better than

'~' equivalent to.

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CHAPTER V

Discussion and Implications

5.0.0. Prologue

It might be recalled that the present study was undertaken with four basic purposes :

(1) to ascertain efficacy of Ausubel strategy;

(2) to ascertain efficacy of Bruner strategy;

(3) to assess relative effectiveness of the two strategies, thereby attempting to throw some light on the long-standing expository-discovery dilemma; and

(4) to estimate differential effects of conceptual style preferences on the acquisition of mathematical concepts under the two strategies.

In order to observe how far these purposes have been served, results obtained from analyses of the data have been discussed in the present chapter. Since the study was conducted in actual classroom situation, it has specific implications for classroom teaching of mathematical concepts. These have been delineated in brief. The study does not claim to furnish final answers to the questions raised. Quite a few inadequacies still remain. While trying to answer some questions, others have cropped up. A few suggestions for further research in this area have, therefore, been put forth.

5.1.0. Efficacy of Ausubel Strategy

It was seen from the results of this study (section 4.16.0.) that Ausubel group performed significantly better ($p < .01$) than control group on 'Concept Knowledge Test' and 'Concept to eleventh grade students, so far as acquiring knowledge of the concepts was concerned. Students taught by Ausubel strategy performed significantly better ($p<.05$) than those taught by Bruner strategy on 'Concept Transfer Test'; whereas, students taught by Bruner strategy performed significantly better ($p<.01$) than those taught by Ausubel strategy on 'Heuristic Transfer Test', 'Short-Term Retention Test' and 'Long Term Retention Test'. Thus, it could be concluded that Ausubel strategy was superior to Bruner strategy in enhancing transfer of the concepts learnt, but for training in transfer of heuristics and for increasing student's ability to retain knowledge of the concepts learnt for short as well as long time, Bruner strategy was superior to Ausubel strategy.

The result of this study, in respect of knowledge criterion, agrees with some of the recent findings. Grabber (1974), King (1976) and Marine (1977) found no significant differences between their treatments when desired outcomes were understanding, identification and immediate recall of the concepts, respectively. For other criteria, however, the results were contrasting. Grabber's (1974) study indicated superiority of 'deductive-expository' method for long-term retention of 'science' concepts; and King (1976) found no significant differences between his treatments on the criteria of 'transfer' and 'delayed retention'.

As has been pointed out earlier (section 2.7.1.), none of these researchers seems to have strictly adhered to the tenets of either Ausubel model or Bruner model. In the present study, not only the employment of full-fledged models (developed into strategies) was planned, but, by analysing data on specially designed 'Observer's Rating Scale', its execution was confirmed. Hence, conclusions of this study with regard to relative efficacy of Ausubel and Bruner models on various criteria could be stated with reasonable confidence. Validity of results on the criterion of 'Concept Transfer' should, however, be held in question, due to relatively low reliability ($r=0.554$) of 'Concept Transfer Test' used in this study.

5.3.1. Searchlight on Expository-Discovery Dilemma

Having accepted the proposition of this study to represent expository method by Ausubel strategy and discovery method by Bruner strategy, it would be interesting to observe some earlier findings about expository-discovery controversy in the light of results obtained from this study.

It has been observed that the results of this study on the criterion of 'concept knowledge' tend to support earlier findings of Loomer (1976), Tate (1978), Jolly (1978), Putt (1979) and Cantor (1982). These researchers found that expository and discovery approaches were equally effective for enhancing students' ability to acquire knowledge of the concepts. Findings of Worthen (1968) and Olander and Robertson (1973) showing superiority of expository approach over discovery approach have not been corroborated by the present study.

On the criterion of 'concept transfer' result of the present study agrees with that of Francis (1975) in which he has shown the superiority of expository approach for the purpose. Worthen (1968), however, indicated that expository and discovery approaches were equally effective for transfer of mathematical concepts. Finding of this study does not support this contention.

Only study that seems to have used 'heuristic transfer' as criterion was that of Worthen (1968) in which he showed superiority of discovery method over expository method for transfer of heuristics. The result of this study corroborates the finding. It further substantiates Suchman's (1964) pointer that "training of inquiry skills in discovery lesson might lead to difference in favour of discovery learning", since such a training was an integral part of Bruner strategy.

In respect of short as well as long term retention, the results of this study are in favour of Worthen (1968) and Olander and Robertson (1973). These investigators found that discovery approach was more effective than expository approach for the purposes. Finding of Loomer (1976) indicating equal effectiveness of the two approaches and that of Francis (1975) showing superiority of expository approach in enhancing retention have not been supported by the present study.

Although agreements and disagreements of the results of present study with those carried out in recent past have been briefly described in the above paragraphs, it must be mentioned here that the studies greatly differed in respect of grade levels and concepts taught so also in treatments, in spite of the fact that they were designated by the common names, expository and discovery. Any generalization, therefore, seemed untenable. But because of careful designing of the present study, it could be said with reasonable confidence that the study highlighted equal effectiveness of the two approaches for knowledge acquisition, superiority of expository approach for transfer and superiority of discovery approach for heuristic transfer and short as well as long term retention, while teaching mathematical concepts of set, relation, function, limit, continuity and derivative to eleventh grade students, provided Ausubel strategy was used as expository method and Bruner strategy as discovery method.

5.4.0 Effects of Conceptual Style Preferences

In order to observe the effects of conceptual syle preferences the data for categorical, descriptive, and relational style subgroups of the groups A, B, C were separately analysed in this study. It was expected that the results of these analyses would reflect on differential effects, if any, of the conceptual style preferences of students on their concept acquisition by the two strategies. It may be recalled that investigation of interactions was beyond the purview of this study since it did not involve factorial experiment.

5.4.1. Impact of Categorical Style Preference

From the results for categorical style subgroups (Table 4.83), it could be seen that superiority of Ausubel group over control group on the criterion of 'concept transfer' was lost by its categorical style subgroup when compared with its counterpart in control group. There was no significant difference between the performance of Ausubel group and control group on long-term retention test but categorical style subgroup of Ausubel group had shown superior performance than its counterpart in control group on long-term retention test.

In respect of comparison between Bruner group and Control group, the results for categorical style subgroups coincided with those for the whole groups on all criterion measures.

Superiority of Ausubel group over Bruner group on the criterion of 'concept transfer' was not maintained by their respective subgroups with categorical style perference.

5.4.2. Impact of Descriptive Style Preference

In comparison with results for the whole groups, descriptive style subgroup of Ausubel group lost superiority over its counterpart in control group on all three criteria: knowledge, transfer, and heuristic transfer. Descriptive style subgroup of Bruner group also lost superiority over its counterpart in control group on the criteria of knowledge and long-term retention. Comparison between descriptive style subgroups of Ausubel and Bruner groups also showed different results from those for the whole groups on the criteria on transfer, short-term retention and long-term retention.

5.4.3. Impact of Relational Style Preference

As compared to results for the whole groups, relational style subgroup of Ausubel group lost superiority over its counterpart in control group on 'transfer' and 'heuristic transfer'. Results for relational style subgroup of Bruner group did not differ from those for the whole groups. Comparison between relational style subgroups of Ausubel and Bruner groups showed different results from those for the whole groups only on 'transfer' criterion.

5.4.4. Differential Effects of the Three Conceptual Style Preferences

From the foregoing description of the impact of different conceptual style preferences on the acquisition of mathematical concepts by Ausubel and Bruner strategies it appears that categorical style subgroup and relational style subgroup of Bruner group produced results which were exactly similar to those for the whole groups on all criteria. Descriptive style subgroup of Bruner group gave different results only on knowledge and long-term retention. This deviation was, perhaps, due to small size (N=5) of the subgroup. The results for the three stylewise subgroups of Bruner group could, thus, be considered to be fairly stable. One is, therefore, inclined to conclude that there was no differential effect of conceptual style preference on the performance of students taught by Bruner strategy.

In respect of Ausubel strategy, however, categorical style subgroup produced slightly better results than the whole group as could be seen from Table 4.83. But descriptive style subgroup of Ausubel group has shown marked declination in performance on various criteria as compared to that of the whole group. This seems to indicate that Ausubel strategy was particularly suitable to the students with categorical style preference. Conceptual style preference, thus, appeared to have differential effect on the performance of students taught by Ausubel strategy. Matching of categorical style preference with Ausubel strategy could, perhaps, be attributed to the very nature of this strategy, in which students are required to link every concept to the advance organizer, or, in other words, to form categories of concepts in a given hierarchy.

Robbinson and Gray (1974) reported that, "inferential (categorical) style was the better predictor of performance on mathematical concepts". The results of this study seem to support this contention to some extent. It should, however, be reiterated that observing differential effect of conceptual style preference on the acquisition of mathematical concepts was among the subsidiary objectives of this study. Findings of this study on this aspect, should, therefore, be treated as tentative in a truly research spirit. Confirmatory evidence could be obtained from a factorial study designed specially for this purpose.

5.5.0. Implications for Teaching

The study seems to hold the following implications for teaching mathematical concepts to eleventh grade students within the frame of experimental setting:

1. Ausubel's Advance Organizer Model and Bruner's Concept Attainment Model should form integral parts of the methodology used for teaching mathematical concepts to eleventh grade students.

2. To the extent that students' ability to acquire knowledge of the concepts is the only valued outcome of instruction, either Ausubel strategy or Bruner strategy could be used equally effectively.

3. To the extent that students' ability to transfer knowledge of the concepts, in similar situation, forms an additional objective of instruction, Ausubel strategy may be preferably used.

4. To the extent that students' ability to acquire knowledge of the concepts and their heuristic transfer in discovering new relationships are the desired objectives of instruction, Bruner strategy may be preferably used.

5. To the extent that students' ability to retain knowledge of the concepts for a short as well as long period of time is an important objective of instruction, Bruner strategy may be preferably used.

6. Students' 'Conceptual Style Preferences' should be taken into account while making the choice of a strategy. Ausubel strategy seems to be useful mainly for categorical style students. Bruner strategy may be used for a mixed class.

5.6.0. Suggestions for Future Research

Following are the few suggestions for future research in this area :

1. Replication of this study should be conducted at other grade levels and for different concepts at the same grade level to test the generalizability of its conclusions.

2. The study may also be replicated for the sample of girl students to observe the effect of sex variable on its results.

3. Effectiveness of Ausubel strategy needs to be checked using more reliable 'Concept Transfer Test'.

4. Efficacy of Ausubel strategy for short as well as long term retention may be retested using advance organizers at more abstract level.

5. While ascertaining the effectiveness of Bruner strategy on the criteria of 'concept transfer' and 'heuristic transfer', carry-over effect of instruction by this strategy should be removed.

6. Separate study may be conducted to test the effectiveness of Bruner strategy using 'selection oriented' and 'unorganized material' forms of Bruner's Concept Attainment Model.

7. Nurturant effects of the two strategies could be studied in addition to instructional effects.

8. Perceptual changes in students on being taught by the two strategies could be studied using semantic differential scale.

9. Process objectives of instruction may be included in a study investigating the effectiveness of the two strategies.

10. Teacher behaviours under the two strategies could be analysed on careful recording and subsequent transcription of the lessons taken by these two strategies.

11. A factorial experiment should be conducted to study interactions of conceptual style preferences with the two strategies of teaching involved in this study.

CHAPTER VI

Summary and Conclusions

6.0.0. The Problem

Instructional strategies devised by mathematics educators to teach mathematical concepts could be broadly classified as: (1) expository and (2) discovery. Expository presentation has been advocated by Ausubel (1968) and to some extent supported by Gagne (1965) and Skemp (1971). Proponents of discovery presentation include Polya (1948), Piaget (1950), Bruner (1966) and Dienes (1963).

Extensive research has been done on the relative efficacy of expository and discovery methods for the teaching of mathematics. But inadequacies pointed out by Wittrock (1966), Bittinger (1968) and Gustafsson (1975) vitiated previous research in this area to such an extent that Cronbach (1966) and Traverse (1973) suggested to put aside this polemic question. One would, however, be inclined to agree with Cantor (1982) when he remarked that, "The problem appears to have sufficient pedagogical and theoretical importance to justify further attempt to shed new light on the matter."

One of the probable reasons for inconclusive research evidence in this area, according to this investigator, was a piecemeal approach adopted by previous researchers. An eclectic approach covering all important dimensions of expository and discovery presentations was, therefore, expected to bring forth some new evidence.

Models of teaching developed by Joyce and Weil (1972) paved the way for conducting such research. Advance Organizer Model of David Ausubel and Concept Attainment

Model of Jerome Bruner stood for choicest representatives of expository and discovery presentations respectively. Weil and Joyce (1978) suggested that these two models were mainly concerned with acquisition and retention of concepts.

Review of research revealed that studies on Ausubel's model produced conflicting evidence about facilitating effect of advance organizer on learning and retention. Brunerian studies also appeared to necessitate further substantiation. A few comparative studies such as those of Grabber (1974), King (1976), and Marine (1977) did not seem to have strictly adhered to the tenets of either Ausbel's model or Bruner's model. Evidence in respect of relative effectiveness of these two models on the acquisition of mathematical concepts was, thus, found wanting. The researcher has, therefore, undertaken the problem of "Evaluating Differential Effectiveness of Ausubel and Bruner Strategies for Acquisition of Concepts in Mathematics".

6.1.0.

Objectives

Following were the principal and subsidiary objectives of the study :

Principal Objectives :

(*i*) To ascertain comparative effectiveness of Ausubel strategy with traditional one on the various criteria of concept acquisition in mathematics.

(*ii*) To ascertain comparative effectiveness of Bruner strategy with traditional one on the various criteria of concept acquisition in mathematics.

(*iii*) To ascertain relative effectiveness of Ausubel and Bruner strategies on the various criteria of concept acquisition in mathematics.

Subsidiary Objectives :

(*i*) To ascertain the effectiveness of Ausubel strategy in the acquisition of concepts in mathematics, separately for the students of different conceptual style preferences.

(*ii*) To ascertain the effectiveness of Bruner strategy in the acquisition of concepts in mathematics, separately for the students of different conceptual style preferences.

(*iii*) To ascertain relative effectiveness of Ausubel and Bruner strategies in the acquisition of concepts in mathematics, separately for the students of different conceptual style preferences.

6.2.0. Sample

The sample consisted of three intact sections of eleventh grade chosen from higher secondary schools of Nagpur city on the basis of their comparability in respect of size, average age, mean-score in mathematics at S.S.C. examination and variability of these scores. Treatments were randomly assigned to these three sections which formed the experimental groups. The group taught by Ausubel strategy contained thirty-seven students; the group taught by Bruner strategy contained thirty-six students; and the control group taught by traditional strategy contained thirty-eight students.

6.3.0. Material

Six concepts were chosen from the mathematics syllabus prescribed by the Maharashtra State Board of Higher Secondary Education for eleventh grade students of science stream. Twenty-five lessons were prepared by each of the three strategies, involving the concepts and their subconcepts. Lessons of Ausubel strategy followed the operational form of Ausubel's Advance Organizer Model; lessons of Bruner strategy followed the operational form of Bruner's Concept Attainment Model; and those of traditional strategy followed traditional method of teaching mathematical concepts in higher secondary schools.

Raven's Standard Progressive Matrices were chosen for the measurement of intelligence. Cognitive Style Test developed by Robbinson and Gray (1974) was suitably adapted by this researcher for the identification of conceptual style preferences. Previous Knowledge Test and all other criterion tests were developed by the researcher. Table 6.1 depicts validity and reliability coefficients of these measures.

In addition to these instruments, one Observer's Rating Scale was developed by the researcher for the measurement of 'Teacher Fidelity' to prescribed treatments. Format designed by Worthen (1970) was adopted for this purpose.

TABLE 6.1

Validity and Reliability Coefficients of the Tests Used

Name of the Test		Validity	Test-Retest Reliability
Raven's Standard Progressive Matrices		.860	.930
Cognitive Style Test	Categorical Style Subtest	.523 (N=42)	.560 (N=37)
	Descriptive Style Subtest	.618 (N=42)	.626 (N=37)
	Relational Style Subtest	.535 (N=42)	.527 (N=37)
Previous Knowledge Test		*	.574 (N=44)
Concept Knowledge Test		*	.802 (N=39)
Concept Transfer Test		*	.554 (N=41)
Heuristic Transfer Test		*	.821 (N=33)
Concept Retention Test I		*	.606 (N=27)
Concept Retention Test II		*	.411 (N=32)

* All these tests had content validity by virtue of their construction on the basis of content outlines judged to be adequate by subject teachers.

6.4.0. Experimental Design

Since random selection of subjects—the requirement of a true experimental design—was difficult to be realized in a classroom experiment, "Non-equivalent Control Group Design" (Campbell and Stanley, 1963, p. 217) was chosen for the study.

6.5.0. The Experiment

Cognitive Style Test and the pre-tests on intelligence and previous knowledge were administered to the three groups immediately on re-opening of the schools after winter vacation of 1981. Experimental teaching commenced on November 12, 1981, and lasted for ten weeks till January 20, 1982. All three groups were taught by the researcher himself strictly according to lesson plans for the three categories. Same concepts were taught for equal time duration to all the three groups. Examples given and assignments set forth were also similar. Each lesson of all the three groups was observed and rated by two previously trained observer-raters : one internal and one external.

Criterion tests I, II, and III, which were respectively made up of first, second, and third parts of concept knowledge test, concept transfer test and heuristic transfer test, were administered to the three groups in the periods following completion of eighth, twentieth, and twenty-fifth lessons respectively. Retention test I was administered exactly after two weeks period and Retention test II was administered exactly after eight weeks period from the completion of experimental teaching to all the three groups.

Answer-sheets of the three groups for all the pre- and post-test measures were scored independently by the researcher and his senior colleague strictly according to the scoring keys prepared in advance. Observer-raters' reports were collected and scrutinized. The agreement between two observer-raters, observing the same lesson simultaniously, were found to be quite high (ranging between .13 and .96) for all the three strategies.

6.6.0. Analyses

The data on Observer's Rating Scale was, first, analysed to assess teacher's fidelity to prescribed teaching strategies. It was found that Ausubel and Bruner strategies were satisfactorily adhered to while teaching mathematical concepts to group A and group B respectively. Treatment given to control group C was found to be contaminated by the characteristics of two experimental strategies. Further scrutiny, however, indicated that this contamination was negligible.

Conceptual Style Preferences of the students in the three groups were identified on the basis of their scores on three subtests of Cognitive Style Test. Ausubel group contained eighteen categorical style students, five descriptive style students and fourteen relational style students; Bruner group contained thirteen categorical style students, six descriptive style students, and seventeen relational style students; and the control group contained sixteen categorical style students, six descriptive style students and sixteen relational style students.

Means and Standard Deviations of the three groups and their style-wise subgroups on pretreatment measures of intelligence and previous knowledge were computed. Descriptive statistics of the three groups and their style-wise subgroups for criterion measures of concept knowledge, concept transfer and heuristic transfer were obtained by pooling the scores on criterion concept tests I, II and III; and those for short-term retention and long-term retention were obtained from the scores on concept retention tests I and II respectively.

Statistical procedure used for inferential analyses was 'Analysis of Covariance' since it was quite possible that the groups and the subgroups might differ on pretreatment measures. Bartlett's test was employed to check the assumption of 'variance homogeneity' before proceeding to analysis of covariance for the data on each criterion measure. Intelligence was taken as constant covariate in the data analyses of all criterion measures. Previous knowledge was an additional covariate while analysing the data on Concept Knowledge Test; whereas, concept knowledge was used as an additional covariate while analysing the data on Concept Transfer Test, Heuristic Transfer Test and Short-Term as well as Long-Term Retention Tests. The scores on Short-Term Retention Test were used as third covariate in the analysis of data on Long Term Retention Test.

All hypotheses were tested against a probability level of .01 to avoid occurrence of Type I error. Those accepted at .01 level were, however, retested at .05 level of significance to avoid Type II error. In addition to F-test, t-test was employed for testing the significance of difference between two adjusted means, where necessary. All analyses were done by the computer of Physical Research Laboratory (P.R.L.), Ahmedabad.

6.7.0. Results

Following results were obtained from inferential analyses which were carried out to test the null hypotheses pertaining to objectives outlined in section 6.1.0. of this summary.

Null hypothesis of mean difference between the performance of groups A and C was rejected ($p<.01$) on the criteria of knowledge and transfer, rejected ($p<.05$) on the criterion of heuristic transfer, and accepted on the criteria of short and long term retention.

Null hypothesis of mean difference between the performance of groups B and C was rejected ($p<.01$) on the criteria of knowledge, heuristic transfer, short-term retention and long-term retention but accepted on the criterion of transfer.

Null hypothesis of mean difference between the performance of groups A and B was rejected ($p<.01$) on the criteria of heuristic transfer, short-term retention and long-term retention, rejected ($p<.05$) on the criterion of transfer, and accepted on the criterion of knowledge.

Null hypothesis of mean difference between the performance of categorical style subgroups of the groups A and C was rejected ($p<.01$) on the criteria of knowledge, and heuristic transfer, rejected ($p<.05$) on the criterion of long-term retention, and accepted on the criteria of transfer and short-term retention; that for descriptive style subgroups of the groups A and C was accepted on all criteria; and that for relational style subgroups of the groups A and C was rejected ($p<.01$) on the criterion of knowledge but accepted on rest of the criteria.

Null hypothesis of mean difference between the performance of categorical style subgroups of the groups B and C was rejected ($p<.01$) on the criteria of knowledge, heuristic transfer, short-term retention and long-term retention but accepted on the criterion of transfer; that for descriptive style subgroups of the groups B and C was rejected ($p<.01$) on the criterion of heuristic transfer, rejected ($p<.05$) on the criterion of short-term retention and accepted on rest of the criteria; and that for relational style subgroups of the groups B and C was rejected ($p<.01$) on the criteria of knowledge, heuristic transfer, short-term retention and long-term retention but accepted on the criterion of transfer.

Null hypothesis of mean difference between the performance of categorical style subgroups of the groups A and B was rejected ($p<.01$) on the criteria of heuristic transfer and short-term retention, rejected ($p<.05$) on the criterion of long-term retention but accepted on the criterion of knowledge and transfer; that for descriptive style subgroups of the groups A and B was rejected ($p<.01$) on the criterion of heuristic transfer but accepted on rest of the criteria; and that for relational style subgroups of the group A and B was rejected ($p<.01$) on the criteria of heuristic transfer, short-term retention and long-term retention but accepted on the criteria of knowledge and transfer.

6.8.0.

Conclusions

Following were some of the major conclusions drawn from the results of this study :

1. Ausubel strategy was superior to traditional strategy for teaching mathematical concepts to eleventh grade students, so far as knowledge, transfer and heuristic transfer of the concepts were concerned.
2. Bruner strategy was superior to traditional strategy for teaching mathematical concepts to eleventh grade students, so far as knowledge, heuristic transfer, short-term retention and long-term retention of the concepts were concerned.
3. Ausubel and Bruner strategies were equally effective for teaching mathematical concepts to eleventh grade students, so far as students' ability to acquire knowledge of the concepts was concerned.
4. Ausubel strategy was superior to Bruner strategy for teaching mathematical concepts to eleventh grade students, so far as enhancing concept transfer was concerned.
5. Bruner strategy was superior to Ausubel strategy for teaching mathematical concepts to eleventh grade

students, so far as students' abilities to transfer heuristics—discover new relationships and to retain knowledge of the concepts learnt for short as well as long period of time—were concerned.

6. Conceptual style preferences of the students seemed to have differential effect on their acquisition of mathematical concepts, when taught by Ausubel strategy. This strategy appeared to be more suitable for teaching mathematical concepts to categorical style students of eleventh grade.

7. Conceptual style preferences of the students did not seem to have differential effect on their acquisition of mathematical concepts, when taught by Bruner strategy. This strategy appeared to be suitable for teaching mathematical concepts to eleventh grade students of all conceptual style preferences.

6.9.0. Implications

The study has following implications for the teaching of mathematical concepts to eleventh grade students :

1. To the extent that students' ability to acquire knowledge of mathematical concepts is the only valued outcome of instruction, either Ausubel strategy or Bruner strategy may be used equally effectively.
2. To the extent that acquiring knowledge of the concepts and their transfer in similar situation are concerned, Ausubel strategy may be preferably used.
3. To the extent that acquiring knowledge of the concepts, their heuristic transfer, and short as well as long term retention are concerned, Bruner strategy may be preferably used.
4. In a mixed class consisting of students with different conceptual style preferences, Bruner strategy may be preferably employed.

6.10.0. Suggestions

Following are the few suggestions for future research in this area :

1. The study should be replicated for different grade levels and subject matters to test the generalizability of its conclusions.
2. Nurturant effects of the two strategies could be studied along with their instructional effects.
3. Perceptual changes in students on being taught by the two strategies could be studied using semantic differential scale.
4. In addition to product objectives process objectives of instruction may be included in a study investigating the effectiveness of these two strategies.
5. Teacher behaviours under the two strategies could be analysed from transcripts of the lessons taken by these two strategies.
6. A factorial experiment should be conducted to study interactions of conceptual style preferences with the two strategies of teaching included in this study.

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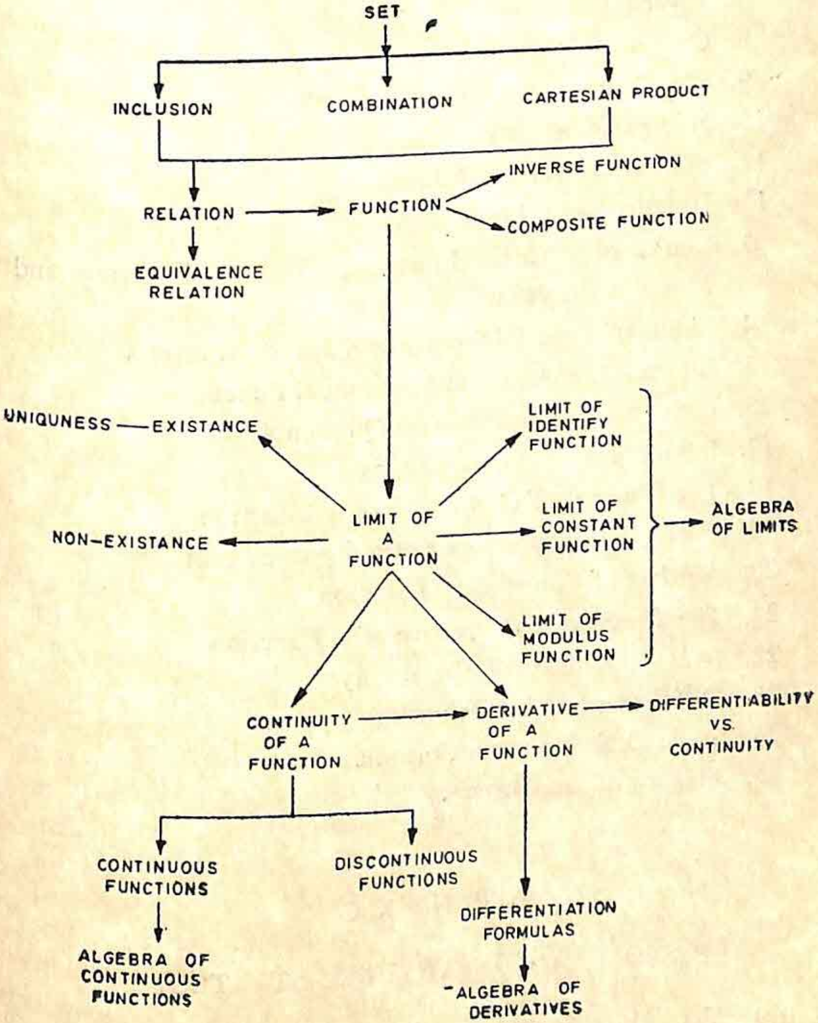
Appendices

APPENDIX A

HIERARCHY OF CONCEPTS



APPENDIX B

UNITS OF TEACHING

1. The Concept of Set
2. Set Inclusion
3. Set Combination
4. Cartesian Product; Relation
5. Equivalence Relation
6. The Concept of Function
7. Inverse Function
8. Composite Function
9. The Concept of Limit of a Function
10. Non-existing Limits
11. Formal Definition of Limit
12. Uniqueness of Limit
13. Limits of Identify Function, Constant Function and Modulus Function
14. Limits of Sum, Difference and Scaler Multiples
15. Limits of Product and Polynomial Functions
16. Limits of Reciprocals and Quotients
17. Limits of Rational Functions
18. The Concept of Continuity of a Function
19. Formal Definition of Continuity at a Point
20. Algebra of Continuous Function
21. The Concept of Derivative of a Function
22. Differentiability Vs. Continuity
23. Differentiation by Definition
24. Some Differentiation Formulas
25. Algebra of Derivatives.

APPENDIX C

LESSON PLAN ON AUSUBEL STRATEGY

Unit 9 : The Concept of Limit of a Function.

I Advance Organizer

1. Essential Characteristics of the Advance Organizer :

   (*i*) An advance organizer should be at higher level of abstraction and generality than the concept of limit.

   (*ii*) It must include the concept of limit.

   (*iii*) It should be capable of providing ideational scaffolding for the concept of limit.

   (*iv*) It must be linked with previous knowledge of the students.

   (*v*) It must strengthen the clarity and stability of cognitive structure.

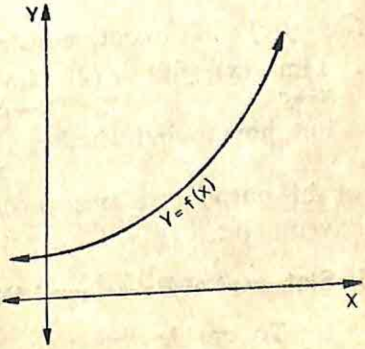
2. Type of Advance Organizer :

An expository organizer must be used since an unfamiliar concept is being introduced.

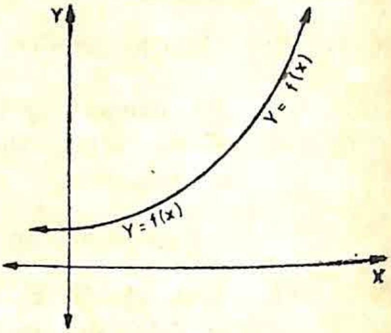
3. Formulation of Advance Organizer Presentation :

The concept of continuity could well be used as an advance organizer since it has all the essential characteristics specified above.

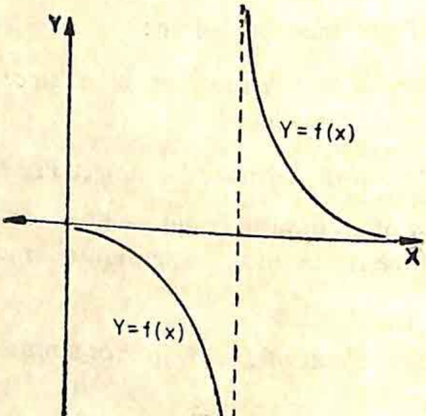
Following is the text of an advance organizer formulated for this lesson.



Consider the graph of a function $y=f(x)$. The graph, as you know, is a curve in Cartesian plane. If this curve goes on without interruption or abrupt change, we say that it is continuous. On the other hand, if there is a gap in the curve at a certain point or it takes sudden jump at a certain point, we say that it is discontinuous at that point. Depending upon whether the graph of a function is continuous or discontinuous at a point, we say that the function is continuous or discontinuous at that point. The question is how to determine analytically the continuity or discontinuity of a function at a given point.



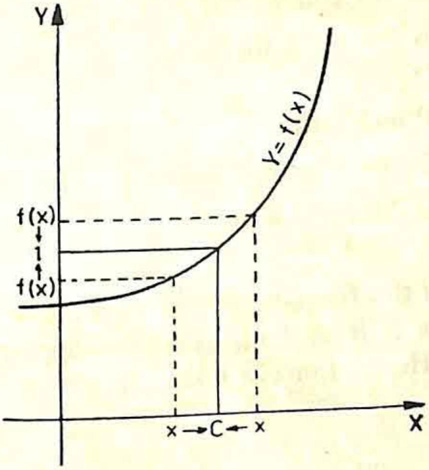
Intuitively, we can say that the function $y=f(x)$ is continuous at $x=c$ iff $f(x)$ approaches $f(c)$ from both sides as x approaches c from the two sides. If $f(x)$ approaches 'l' from both sides as x approaches 'c' from the two sides and $l \neq f(c)$, then the function is discontinuous at $x=c$. The 'l', here, is called limit of the function as x tends to c and we write : $\lim\limits_{x \to c} f(x)=l$. f is continuous at $x=c$, iff $\lim\limits_{x \to c} f(x)=f(c)$ and discontinuous at $x=c$, either if (1) $\lim\limits_{x \to c} f(x) \neq f(c)$ or (2) $\lim\limits_{x \to c} f(x)$ does not exist.



But, how to find whether $\lim\limits_{x \to c} f(x)$ exists or does not exist. For this purpose we must precisely define limit of a function at a given point.

II. Statement of Behavioural Objectives

1. To enable the students to formally define the limit of a function.



2. To enable them to identify whether the limit of a function at a given point exists or does not exist.

3. To enable them to assimilate the concept of limit in their cognitive structure.

III. Presentation of Learning Task

The meaning of $\mathop{\mathrm{Lim}}\limits_{x \to c} f(x)=1$, as seen earlier, is that $f(x)$ approaches 1 from both sides as x approaches c from the two sides. But this description of limit is too vague to be called mathematics. So we need a more precise definition of the limit of a function.

To say that $\mathop{\mathrm{Lim}}\limits_{x \to c} f(x)=1$ is to say that the difference between $f(x)$ and 1 can be made arbitrarily small (smaller than any pre-assigned positive number) simply by requiring that x be sufficiently close to c.

Now, think of a positive number and call it ε (epsilon). If it is true that $\mathop{\mathrm{Lim}}\limits_{x \to c} f(x)=1$, then we can be sure that, $|f(x)-1| > \varepsilon$ for all x sufficiently close to c, namely, we can be sure that there exists a positive number δ (delta) such that if $0 < |x-c| < \delta$,

then $\quad |f(x)-1| < \varepsilon$.

Thus, we formally define limit as :

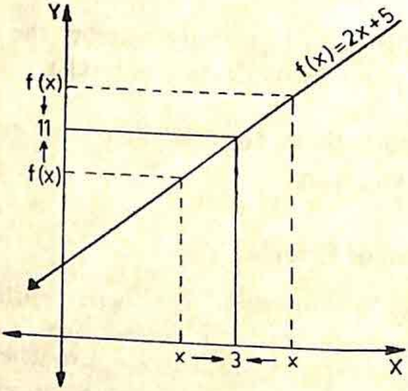
$$\lim_{x\to c} f(x)=l \text{ iff for each } \varepsilon>0,$$

there exists $\delta>0$ such that,

if $\quad 0<|x-a|<\delta, \quad$ then $\quad |f(x)-l|<\varepsilon.$

Now consider $\lim\limits_{x\to 3} (2x+5)$

The graph of this function, as you know, is a straight line. As x approaches 3 from both sides, $f(x)$ approaches *11* from the two sides. Hence, $\lim\limits_{x\to 3}(2x+5)=11.$



Analytically, Let $\varepsilon>0$ be assigned. We seek a number $\delta>0$ such that if

$$0<|x-3|<\delta,$$

then $\quad |(2x+5)-11|<\varepsilon,$

we note that, $\quad |(2x+5)-11|<\varepsilon$

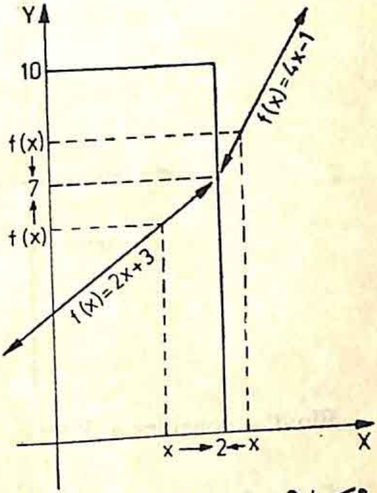
$$\begin{aligned}&\Leftrightarrow |2x-6|<\varepsilon\\&\Leftrightarrow 2|x-3|<\varepsilon\\&\Leftrightarrow |x-3|<\varepsilon/2\end{aligned}$$

Thus, if we choose $\delta=\varepsilon/2$, we find that

$$0<|x-3|<\delta \Rightarrow |(2x+5)-11|<\varepsilon$$

i.e., $\quad \lim\limits_{x\to 3} (2x+5)=11.$

Codsider a function $f(x)=\begin{cases} 2x+3 \text{ when } x<2 \\ 4x-1 \text{ when } x>2 \\ 10 \text{ when } x=2 \end{cases}$



We want to evaluate

$$\lim_{x\to 2} f(x)$$

From the graph of the function it could be observed that $f(x)$ approaches 7 as x approaches 2 from left and $f(x)$ approaches 7 as x approaches 2 from right, irrespective of the fact that $f(2)=10$.

Thus, $\lim_{x\to 2} f(x)=7$

Now Let $\varepsilon>0$ be assigned, we show that there exists $\delta>0$ such that if $0<|x-2|<\delta$ then $|f(x)-7|<\varepsilon$

For $x<2$, $|f(x)-7|=|(2x+3)-7|=|2x-4|$
$=2|x-2|$

For $x>2$, $|f(x)-7|=|(4x-1)-7|=|4x-8|$
$=4|x-2|$

Thus, when x approaches 2 from left,

taking $\delta=\varepsilon/2$ we find that

$$0<|x-2|<\delta \Rightarrow |f(x)-7|<\varepsilon$$

and when x approaches 2 from right,

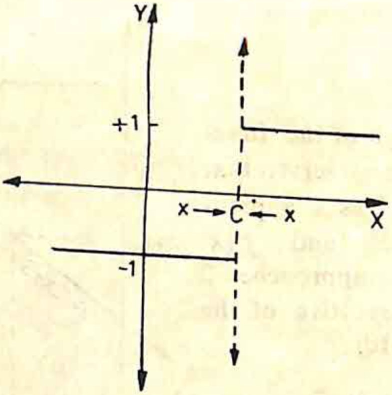
taking $\delta=\varepsilon/4$ we find that

$$0<|x-2|<\delta \Rightarrow |f(x)-7|<\varepsilon$$

Hence $\lim_{x\to 2} f(x)=7$.

Now consider a function $f(x)=\begin{cases} -1, x<c \\ +1, x>c \end{cases}$

As x approaches c from left $f(x)$ is constantly -1 and as x approaches c from right $f(x)$ is constantly $+1$. In this case $\operatorname*{Lim}_{x\to 0} f(x)$ does not exist.

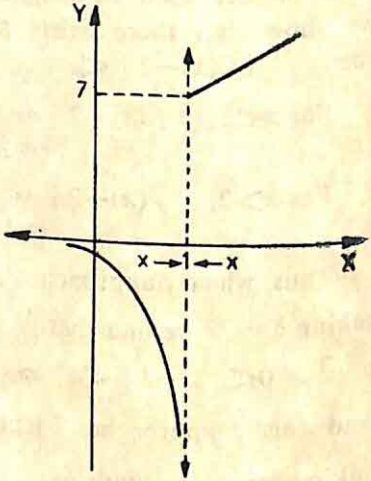


Finally, consider a function

$$f(x)=3x+4 \quad \text{when } x\geqslant 1$$

$$=\frac{1}{x-1} \quad \text{when } x<1.$$

As x approaches 1 from right, $f(x)$ approaches 7; but as x approaches 1 from left, $f(x)$ becomes exceedingly smaller (smaller than any preassigned negative number). In this case also $\operatorname*{Lim}_{x\to 1} f(x)$ does not exist.



IV. Strengthening Cognitive Structure

Now observe how this concept of limit helps us to determine whether the function is continuous at a point or not.

In example 1, $\operatorname*{Lim}_{x\to 3} (2x+5)=11$

Also, $(2x+3)_{x=3}=11$

Thus, $\lim_{x \to 3} f(x) = f(3)$

Hence, the function $f(x) = (2x+5)$ is continuous at $x=3$.

In example 2, $\lim_{x \to 2} f(x) = 7$ but, $f(2) = 10$.

Thus, $\lim_{x \to 2} f(x) \neq f(2)$

Hence, the function $f(x)$ in this example is discontinuous at $x=2$.

In example 3, $\lim_{x \to c} f(x)$ does not exist.

Hence, this function is discontinuous at $x=c$.

Similarly, in example 4, $\lim_{x \to 1} f(x)$ does not exist.

Hence, this function is discontinuous at $x=1$.

The concept of limit is, thus, closely linked with the concept of continuity.

V. Description of Teacher's Activity in Each Phase of the Strategy

Phase 1. Presentation of Advance Organizer : The teacher presents the advance organizer by illustrating it with the help of diagrams drawn on the blackboard.

Phase 2. Presentation of Concept by Definition : The teacher writes definition of limit of a function on the blackboard and tries to bring about its meaningful understanding by heming the discussion around it.

Phase 3. Illustration of Concept by Examples : The teacher illustrates the definition of limit by examples. In doing so he wields full control over the situation and acts as primary source of knowledge.

Phase 4. Relation of Concept of Advance Organizer : The teacher relates the concept of limit to an advance organizer by showing its close connection with the concept of continuity.

APPENDIX D

LESSON PLAN ON BRUNER STRATEGY

Unit 9 : The Concept of Limit of a Function.

I. Analysis of the Concept

1. Name of the concept
   Limit of a function $y=f(x)$
   as x tends to c.
2. Attributes of the concept
   (*a*) Essential attributes :
      (*i*) Tendency of $f(x)$ to approximate '1' as 'x' approaches 'c' from left
      (*ii*) Tendency of $f(x)$ to approximate '1' as 'x' approaches 'c' from right
   (*b*) Non-essential attributes :
      (*i*) The type of function $y=f(x)$
      (*ii*) Existence of $f(x)$ at $x=c$.
3. Type of the concept

Since the concept is defined by joint presence of its essential attributes, it is a conjunctive concept.

II. Statement of Behavioural Objectives

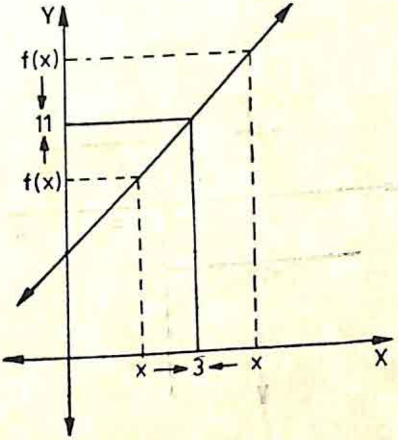
1. To enable the students to attain the concept of limit from examples and non-examples.
2. To enable them to formally define the concept of limit.
3. To enable them to analyse their concept attainment strategies.

III. Presentation of Exemplars and Non-Exemplars

Since reception oriented concept attainment model has been chosen for teaching the concept, exemplars and non-exemplars would be presented in a prearranged order, labelling them "yes" or "no" as the case may be. Concept involved being conjunctive type, exemplars ('yes' examples) would contain all the essential attributes of the concept. In the non-exemplars ('no' examples), on the other hand, one or more of the essential attributes would be absent. Non-essential attributes would serve the purpose of producing noise which is necessary for proper attainment of the concept. Following is the text for the presentation of exemplars and non-exemplars.

Here are some statements involving the concept of limit of a function. Those which illustrate the concept have been marked "yes" and those which do not have been marked "no". Study the exemplars and non-exemplars carefully and identify the concept involved in the exemplars.

1. Consider a function $f(x)=2x+5$

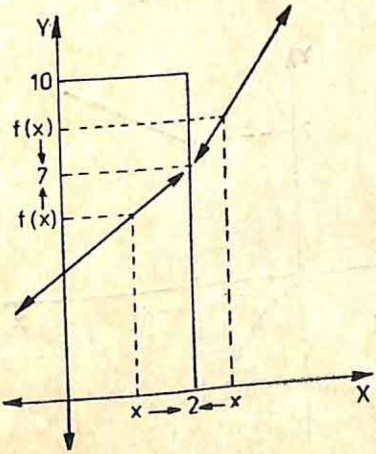


YES

As x approaches 3 from left, $f(x)$ approaches 11

As x approaches 3 from right, $f(x)$ approaches 11

2. Consider a function $f(x)=\begin{cases} 2x+3 \text{ when } x<2 \\ 4x-1 \text{ when } x>2 \\ 10 \text{ when } x=2 \end{cases}$

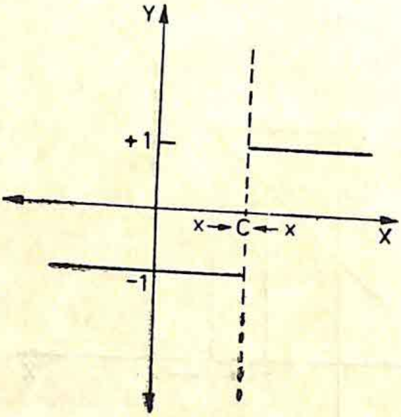


YES

$f(2)=10$

But, as x approaches 2 from left, $f(x)$ approaches 7 and as x approaches 2 from right, $f(x)$ again approaches 7.

3. Consider a function $f(x)=\begin{cases} -1, & x<c \\ +1, & x>c \end{cases}$

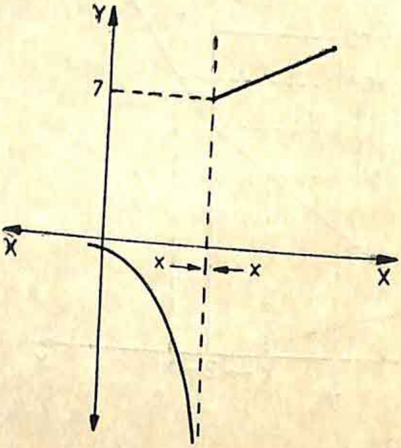


NO

As x approaches c from left, $f(x)$ is constantly -1 and as x approaches c from right, $f(x)$ is constantly $+1$.

4. Consider a function

$$f(x)=3x+4 \text{ when } x\geqslant 1$$
$$=\frac{1}{x-1} \text{ when } x<1$$

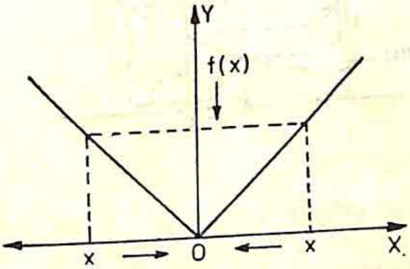


NO

As x approaches 1 from right,
$f(x)$ approaches 7

As x approaches 1 from left, $f(x)$ becomes exceedingly smaller (smaller than any preassigned negative number)

5. Consider a function
$f(x) = |x|$



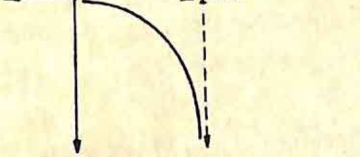
YES

As x approaches 0 from left,
$f(x) = |x|$ approaches 0

As x approaches 0 from right,
$f(x) = |x|$ approaches 0

6. Consider a function

$$f(x) = \frac{1}{x-2}$$



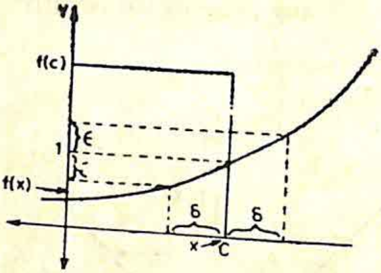
NO

As x approaches 2 from left, $f(x)$ becomes exceedingly smaller (smaller than any preassigned negative number)

As x approaches 2 from right, $f(x)$ becomes exceedingly larger (larger than any preassigned positive number)

7. Consider a function

$$y=f(x)$$



YES

For every arbitrarily chosen $\varepsilon<0$,

there exists $\delta>0$,

such that

if $0<|x-c|<\delta$

then $|f(x)-1|<\varepsilon$

IV. Concept Attainment Strategies

Under instruction by reception oriented concept attainment model the students may use reception strategies such as (*i*) wholist ; or (*ii*) partist for identifying concept embedded in exemplars and non-examplars. Those using wholist strategy would take first "yes" exemplar as concept hypothesis and compare its attributes with subsequent exemplars and non-exemplars to finally identify the concept. The partist would formulate their concept hypothesis only on the part of first "yes" exemplar and confirm or revise it on encountering subsequent exemplars and non-exemplars.

V. Description of Opening Moves of the Teacher in Each Phase of the Strategy

Phase 1. Presentation of Positive and Negative Examples: The teacher would present the sequence of positive and negative examples by writing them down on the blackboard, Positive examples (exemplars) would be labelled as "Yes" and negative examples (non-exemplars) as "No".

Phase 2. Identification of Concept by the Students: The teacher would ask the students to observe carefully the exemplars and non-exemplars and hypothesize about the concept embedded in them. As the students start mentioning their hypotheses he would go on recording them. A prompt would be given if necessary and additional examples supplied if demanded. Balancing the hypotheses put forth by the students, he would enable them to identify the concept.

Phase 3. Elicitation of Definition from the Students: The concept having been identified by the students as: "$f(x)$ approaches the same number '1' as x approaches 'c' from both sides", the teacher would name the concept as "limit of the function as x tends to c" and symbolize it as:

$$\lim_{x \to c} f(x) = 1$$

He would then attract students' attention to the last exemplar in the series and elicit formal definition of the concept of limit from them. Having defined the concept as:

$$\lim_{x \to c} f(x) = 1$$

iff for every $\varepsilon > 0$, $\exists\ \delta > 0$ such that

$$0 < |x - a| < \delta \Rightarrow |f(x) - 1| < \varepsilon$$

he would apply the definition to one or two exemplars and illustrate the $\epsilon - \delta$ technique.

Phase 4. Analysis of Concept Attainment Strategies: The teacher would ask the students to verbalize their thinking process in identifying the concept from exemplars and non-exemplars. He would then analyse the strategies used by them and reveal the nature of each strategy. It would be emphasized that no strategy should be held as superior to another and that the students should always employ their own preferred strategies while attaining the concepts.

APPENDIX E

LESSON PLAN ON TRADITIONAL STRATEGY

Unit 9: The Concept of Limit of a Function.

I. Introduction

1. Recapitulation of Previous Knowledge :

You have learnt earlier the important concepts of relation and function. Relation from set A to set B was seen to be a rule of correspondence between the two sets and was defined as a subset of cartesian product AXB, satisfying the condition involved in the rule.

A rule of correspondence which associated every element of set A with one and only one element of set B was called function. Thus every function was seen to be a relation and as such a set of ordered pairs which when plotted in a cartesian frame gave the graph of a function.

2. Naming the concept :

In today's lecture you are going to learn the concept of limit of a function at a given point.

3. Importance of the Concept :

With the introduction of the concept of limit you are beginning the study of an important branch of mathematics—the calculus. The knowledge of this concept will further help you to understand the notions of continuity of a function and derivative of a function.

II. Statement of Behavioural Objectives

1. To enable the students to define the concept of limit of a function.
2. To enable them to memorize the definition.
3. To enable them to use the definition in solving simple mechanical problems.

III. Presentation of Definition

Let $y=f(x)$ be a function. If $f(x)$ approaches '1' as x approaches c from both sides, we say that '1' is the limit of $f(x)$ as x tends to c. We write this as :

$$\lim_{x \to c} f(x) = 1$$

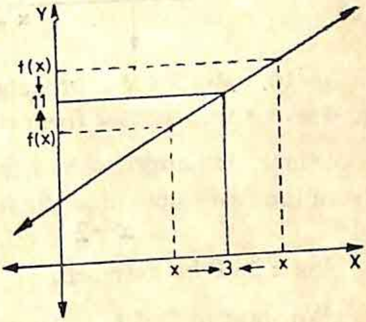
Formal definition of limit is as follows :

$$\lim_{x \to c} f(x) = 1$$

iff for every $\varepsilon > 0$, there exists $\delta > 0$, such that $0 < |x-c| < \delta$ implies $|f(x)-1| < \varepsilon$.

IV. Exemplification of Definition

Consider a function $f(x)=2x+5$. You know that this is a linear function. Hence, its graph will be a straight line in cartesian plane.



From the graph, you can see that as x approaches 3 from the two sides, $f(x)$ approaches 11. Hence, $\lim_{x \to 3} (2x+5)=11$

Now assign an $\varepsilon > 0$, we want to show that there exists $\delta > 0$, such that

$$0 < |x-3| < \delta \Rightarrow |(2x-5)-11| < \varepsilon$$
$$|(2x+5)-11| = |2x-6| = 2|x-3|$$
$$2|x-3| < \varepsilon \Leftrightarrow |x-3| < \varepsilon/2$$

So we take $\qquad \delta=\varepsilon/2$

Now if $\quad |x-3| < \delta = \varepsilon/2$

then $\qquad 2|x-3| < \varepsilon$

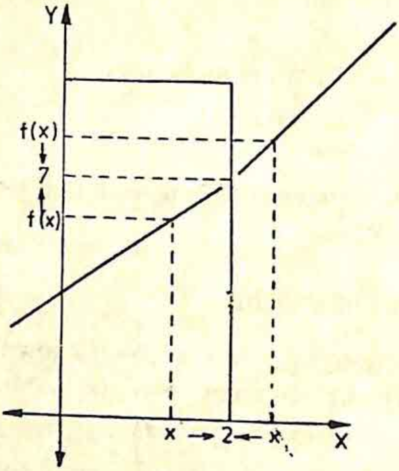
i.e. $\qquad |2x-6| < \epsilon$

i.e. $\qquad |(2x+5)-11| < \varepsilon$

Hence, $\qquad \lim_{x \to 3} (2x+5)=11.$

Now consider a function

$$\begin{aligned} f(x) &= 2x+3, \text{ when } x < 2 \\ &= 4x-1, \text{ when } x > 2 \\ &= 10, \quad \text{ when } x = 2 \end{aligned}$$



$f(2)=10$. But As x approaches 2 from left, $f(x)$ approaches 7 and as x approaches 2 from right, $f(x)$ approaches 7.

Since, $f(x)$ approaches 7 from both sides as x approaches 2 from the two sides $\lim\limits_{x\to 2} f(x)=7$.

Let $\varepsilon > 0$ be assigned.

We want to find $\delta_s > 0$ such that

$$0 < |x-2| < \delta \Rightarrow |(2x+3)-7| < \delta$$

and

$$0 < |x-2| < \delta \Rightarrow |(4x-1)-7| < \varepsilon$$

$$|(2x+3)-7| = |2x-4| = 2\,|x-2|$$

and

$$|(4x-1)-7| = |4x-8| = 4\,|x-2|$$

Thus, when x approaches 2 from left,

we can take $\delta=\varepsilon/2$

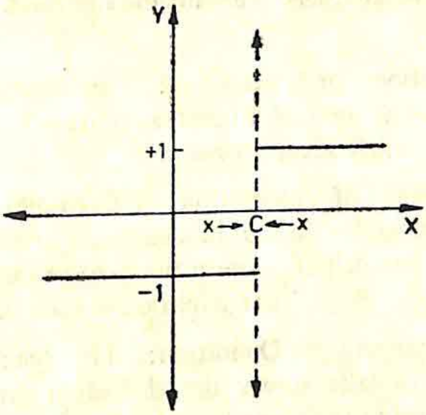
and, when x approaches 2 from right,

we can take $\delta=\varepsilon/4$

Since δ_s exist, we have $\lim\limits_{x\to 2} f(x)=7$

Consider a function $f(x)=\begin{cases} -1, & x < c \\ +1, & x > c \end{cases}$

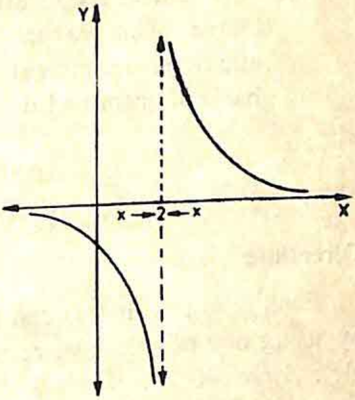
From the graph of this function, we observe that $f(x)$ is constantly -1 when x approaches c from left and $f(x)$ is constantly $+1$ when x approaches c from right.



In this case, we say that $\lim_{x \to c} f(x)$ does not exist.

Finally, consider a function $f(x)=\frac{1}{x-2}$.

The graph of this function comes out as given below:



As x approaches 2 from left, $f(x)$ becomes exceedingly smaller (smaller than any preassigned negative number) and as x approaches 2 from right, $f(x)$ becomes exceedingly larger (larger than any preassigned positive number).

Hence, $\lim_{x \to 2} \frac{1}{x-2}$ does not exist.

Find yourself whether the following limits exist :

(*i*) $\lim_{x \to 1} f(x)$, $f(x)=\begin{cases} 3x+4, & \text{when } x \geqslant 1 \\ \frac{1}{x-1}, & \text{when } x < 1 \end{cases}$

(*ii*) $\lim_{x \to 0} |x|$

(*iii*) $\lim_{x \to 3} f(x)$, $f(x)=\begin{cases} 10, & \text{when } x>3 \\ 2x+3, & \text{when } x<3 \end{cases}$

V. Description of Teachers' Verbalization in Each Phase of the Strategy

1. Presentation of Definition: The teacher writes the definition of limit of a function on the blackboard and reads it loudly several times.
2. Illustration of Definition by Examples: The teacher writes examples on the blackboard, draws figure and explains the definition with the help of the problem and the figure. Pupils' participation is minimal.
3. Memorization of Definition: The teacher asks the students to take down the definition written on the blackboard in their notebook. He then advises them to read the definition several times till it is committed to their memory.He also tells them that if necessary they may write the same definition several times in order to facilitate memorization. Finally, he asks one or two students in the class to stand and recite the definition.
4. Mechanical Application of Definition to Solve Problems: The teacher solves problems on the blackboard eliciting mechanical response from the students on the basis of crammed definition.

APPENDIX F

COGNITIVE STYLE TEST

Directions

Each item in this test can be made a meaningful statement by using one of the three responses in parenthesis. For example: Item X below reads "Dog is to Horse as Crow is to (*a*) Tree (*b*) Black (*c*) Parrot". The correct answer is 'Parrot' because just as dog and horse are animals, crow and parrot are birds. In your answer-sheet, therefore, you will encircle '*c*' against X as follows :

X	*a*	*b*	ⓒ

In item Y, the correct answer is Filament, so you will encircle '*b*' against Y. Similarly '*a*' against Z will be encircled.

X. Dog : Horse :: Crow : (*a*. Tree *b*. Black *c*. Parrot)

Y. Lamp : Wick :: Bulb : (*a*. Electricity *b*. Filament *c*. Switch)

Z. Calf : Cow :: Child : (*a*. Mother *b*. Father *c*. Home)

Read each item carefully. Decide a category or relation between the first two terms. Associate the third term with one of the three responses in bracket in order that a category or relation is established between the third and the fourth term similar to that between the first two.

Your answers are to be recorded on a special answer sheet. Please do not make any mark on this test blank. Fill out the headings on the answer-sheet before starting the test. Do not begin unless you are told to do so.

I

1. Ganga : Kaveri :: Himalaya : (*a*. Everest *b*. Gangotri *c*. Sahyandri)
2. Rose : Chameli :: Orange : (*a*. Garden *b*. Mango *c*. Potato)
3. August : September :: Winter : (*a*. Spring *b*. Summer *c*. October)
4. Rice : Weat :: Cotton : (*a*. Jute *b*. Cloth *c*. Wool)
5. Hands : Legs :: Eyes : (*a*. Spectacles *b*. Vision *c*. Ears)
6. Tea : Coffee :: Goldspot : (*a*. Milk *b*. Beer *c*. Campacola)
7. Cricket : Hockey :: Chess : (*a*. Bridge *b*. Kabaddi *c*. Badminton)
8. Bangles : Necklace :: Snow : (*a*. Mirror *b*. Face *c*. Powder)
9. Bicycle : Scooter :: Boat : (*a*. Launch *b*. Train *c*. Plane)
10. Fish : Meat :: Carrot : (*a*. Eggs *b*. Brinjal *c*. Water)
11. Europe : America :: Maharashtra : (*a*. Asia *b*. Bombay *c*. Kerala)

12. Chair : Table :: Mug : (*a.* Dish *b.* Tea *c.* Stool)
13. Copper : Silver :: Carbon : (*a.* Pencil *b.* Sulphur *c.* Black)
14. Test-Tube : Flask :: Books : (*a.* Shelf *b.* Story c. Magazine)
15. Square : Circle :: Prism : (*a.* Sphere *b.* Spectrum *c.* Glass)

II

1. Lead : Pencil :: Ink : (*a.* Bottle *b.* Paper *c.* Pen)
2. Leather : Shoe :: Wood : (*a.* Saw *b.* Almirah *c.* Tree)
3. Lense : Camera :: Eye : (*a.* Body *b.* Spectacle *c.* Vision)
4. Chair : Legs :: Flag : (*a.* Colour *b.* Post *c.* Nation)
5. Drama : Stage :: Cinema : (*a.* Music *b.* Hero *c.* Screen)
6. Car : Wheels :: Bird : (*a.* Tree *b.* Sky *c.* Wings)
7. Fish : Aquarium :: Animal : (*a.* Jungle *b.* Zoo *c.* Farmer)
8. Picture : Colour :: Poem : (*a.* Poet *b.* Book *c.* Words)
9. Athlete : Physique :: Scholar : (*a.* Knowledge *b.* Habits *c.* Books)
10. Horse : Reins :: Human : (*a.* Life *b.* Senses *c.* Kindness)
11. Sugar : Sweetness :: Music : (*a.* Guitar *b.* Lyric *c.* Melody)
12. Flower : Fragrance :: Man : (*a.* Woman *b.* Character *c.* Wealth)
13. Lamp : Light :: Learned : (*a.* Fame *b.* Degree *c.* Position)
14. Boat : Rudder :: Life : (*a.* Longevity *b.* Aim *c.* Success)
15. Geometry : Axioms :: Nation : (*a.* Population *b.* Prosperity *c.* Constitution)

III

1. Father : Mother :: Uncle : (*a.* Cousin *b.* Aunt *c.* Sister)
2. Pencil : Paper :: Chalk : (*a.* Teacher *b.* Calcium *c.* Blackboard)
3. Milk : Cow :: Egg : (*a.* Hen *b.* Proteins *c.* Omlete)
4. Ship : Harbour :: Bus : (*a.* Conductor *b.* Driver *c.* Station)

5. Coal : Locomotive :: Petrol : (*a.* Fire *b.* Scooter *c.* Dry-cleaner)
6. Client : Lawyer :: Patient : (*a.* Disease *b.* Doctor *c.* Hospital)
7. Flower : Bees :: Sugar : (*a.* Crystal *b.* Water *c.* Ants)
8. Wire : Electricity :: Tap : (*a.* Water *b.* Basin *c.* Meter)
9. Moon : Earth :: Earth : (*a.* Ocean *b.* Sun *c.* Space)
10. Pupil : Teacher :: Public : (*a.* Reformer *b.* Government *c.* Peace)
11. Portrait : Painter :: Book : (*a.* Printer *b.* Publisher *c.* Author)
12. Shadow : Light :: Ignorance : (*a.* Ego *b.* Knowledge *c.* Tolerance)
13. Ebbs : Tides :: Despair : (*a.* Hope *b.* Anxiety *c.* Faith)
14. Flower : Thorn :: Joy : (*a.* Insult *b.* Injury *c.* Sorrow)
15. Soul : Body :: Logic : (*a.* Science *b.* Philosophy *c.* Mathematics)

Answer Sheet

Name of the Student : ....

Age : .... Yrs. .... Months. Sex : Male/Female

Name of the Institution : ....

Class : .... Section : .... Date : ....

Time : From .... To .... Duration : ....

No.	I			II			III		
1	a	b	c	a	b	c	a	b	c
2	a	b	c	a	b	c	a	b	c
3	a	b	c	a	b	c	a	b	c
4	a	b	c	a	b	c	a	b	c
5	a	b	c	a	b	c	a	b	c
6	a	b	c	a	b	c	a	b	c
7	a	b	c	a	b	c	a	b	c
8	a	b	c	a	b	c	a	b	c
9	a	b	c	a	b	c	a	b	c
10	a	b	c	a	b	c	a	b	c
11	a	b	c	a	b	c	a	b	c
12	a	b	c	a	b	c	a	b	c
13	a	b	c	a	b	c	a	b	c
14	a	b	c	a	b	c	a	b	c
15	a	b	c	a	b	c	a	b	c

Scoring Key

Item No.	Scale I (Categorical)	Scale II (Descriptive)	Scale III (Relational)
1	c	c	b
2	b	b	c
3	b	a	a
4	a	b	c
5	c	c	b
6	c	c	b
7	a	b	c
8	c	c	a
9	a	a	b
10	b	b	a
11	c	c	c
12	a	b	b
13	b	a	a
14	c	b	c
15	a	c	c

APPENDIX G

PREVIOUS KNOWLEDGE TEST

Name of the Student :

... ...

Age : ...... Yrs. ...... Months.

Sex : Male/Female

Name of the Institution :

... ...

Class : .... Section : ......

Date : ... ...

Time : From ....To.........

Duration : ... ...

Dear Student,

Here is a simple test of your understanding of the concepts which you learnt in previous two grades. Please answer all questions to the best of your ability. Do not spend undue time on any single item. Your answers are to be written in the space provided for the purpose on this sheet itself.

N.B. : Choose the correct alternative. **Answers**

1. The square root of 3 is ... ...
   (A) odd (B) prime (C) irrational (D) imaginary

2. $\bar{a}^2=?$ ...
   (A) —2a (B) $\frac{a}{2}$ (C) $-a^2$ (D) $\frac{1}{a^2}$

3. The degree of a polynomial $3x^2-4x+5$ is ... ...
   (A) 3 (B) 2 (C) —4 (D) 5

4. If $a \neq b$ and $b \nless a$, then ... ...
   (A) $b<a$ (B) $a<b$ (C) $a \nless b$ (D) $b=a$

5. Which of the following is an open sentence ? ...
   (A) $16+9=25$ (B) $16-9=25$ (C) $x+9=25$
   (D) Add 9 to 16

6. The condition for $\frac{x^2-1}{x+2}$ to be a rational algebraic expression is... ... ...
   (A) $x=1$ (B) $x \neq 1$ (C) $x=-2$ (D) $x \neq -2$

7. If a, b, c are in continued proportion, then ... ...
   (A) a^2-bc (B) $b^2=ac$ (C) $c^2=ab$ (D) $abc=1$

8. The vertex of a parabola $y=x^2-6x+9$ is ... ...
   (A) (0, 3) (B) (3, 0) (C) (0, —3) (D) (—3, 0)

9. The graph of $x-2 \leqslant 7$ is ... ...
   (A) a line (B) a ray (C) a segment (D) a point

10. If $|a| < |b|$, then ...
(A) $a<b$ (B) $-a<-b$ (C) $a^2<o^2$ (D) $b^2<a^2$

11. If a^x-y, $a>0$ and $a\neq1$, then ... ...
(A) $a=\log_x y$ (B) $a=\log_y x$ (C) $x=\log_a y$
(D) $x=\log_y a$

12. The equation $ax^2+bx+c=0$, $a\neq0$ has equal roots ...
if (A) $b^2-4ac>0$ (B) $b^2-4ac=0$ (C) $b^2-4ac<0$
(D) b^2-4ac is a perfect square.

APPENDIX H

CONCEPT KNOWLEDGE TEST

Name of the Student:
... ...
Age : ......Yrs......Months.
Sex : Male/Female
Name of the Institution :
... ...
Class : .... Section ....
Date : ... ...
Time : From ... ... To ......
Duration : ... ...

Dear Student,

Here is a simple test of your knowledge of the concepts learnt during experimental teaching. Please answer all questions to the best of your ability. Do not spend undue time on any single item. Your answers are to be written in the space provided on this sheet itself.

Answers

N.B. Choose the correct alternative

PART I

1. Which of the following collections is not a set ?
   (A) Collection of books in the school library.
   (B) Collection of all beautiful things.
   (C) Collection of birds without wings. ...
   (D) Collection of all natural numbers.

2. Which of the following sets of ordered pairs determine the relation "is less than" from.
   A to B where A={0, 1} and B={1, 2} ?
   (A) {(0, 1), (1, 2), (2, 1)}
   (B) {(0, 1), (1, 1), (1, 2)}
   (C) {(0, 1), (0, 2), (1, 2)} ...
   (D) {(0, 1), (0, 2), (1, 1), (1, 2)}

3. Which of the following associations determine a function $f : X \rightarrow Y$ where $X = \{1, 2, 3, 4\}$ and $Y = \{a, c, e\}$

(A) $1 \rightarrow a$	(B) $1 \rightarrow c$	(C) $1 \rightarrow e$	(D) None of
$2 \rightarrow c$	$2 \rightarrow e$	$2 \rightarrow c$	A
$3 \rightarrow e$	$3 \rightarrow c$	$3 \rightarrow a$	B
	$4 \rightarrow e$	$4 \rightarrow e$	C
		$3 \rightarrow c$	...

4. Which of the following functions has an inverse ?
   (A) $f(X) = |X|$, $X \varepsilon I$
   (B) $f : A \rightarrow B$, such that $f(X) = 3X$, where $A = \{-2, -1, 0, 2\}$, $B = \{-6, -3, 0, 3, 6\}$
   (C) $f(X) = 2X + 3$, XER
   (D) $f : A \rightarrow B$ defined by the set $\{(\alpha, X), (\beta, X), (\gamma, Y)\}$... where $A = \{\alpha, \beta, \gamma\}$ and $B = \{X, Y, Z\}$

PART II

5. $\lim\limits_{x \to c} f(x) = 1$ implies
   (A) f is defined at c
   (B) f is defined at some point near c
   (C) $f(x)$ approaches 1 as x approaches c from both sides.
   (D) $f(c) = 1$

6. $\lim\limits_{x \to c} f(x)$ does not exist if
   (A) f is not defined at c
   (B) f is defined at c but its graph is broken at c
   (C) f is a constant function
   (D) $f(x) = \dfrac{1}{x - c}$

7. $\lim\limits_{x \to c} f(x) = 1$ if and only if
   (A) for every $\delta > 0$, there exists $\varepsilon > 0$ such that, $0 < |x - c| \Rightarrow |f(x) - 1| < \varepsilon$
   (B) for every $\varepsilon > 0$, there exists $\delta > 0$ such that, $0 < |x - c| < \delta \Rightarrow |f(x) - 1| < \varepsilon$
   (C) for every $\varepsilon > 0$, there exists $\delta > 0$ such that, $|x - c| < \delta \Rightarrow |f(x) - 1| < \varepsilon$

(D) for every $\varepsilon>0$, there exists $\delta>0$ such that,
$|f(x)-1| < \varepsilon \Rightarrow |x-c| < \delta$ ...

8. A function f is continuous at c iff
   (A) for x close to c, $f(x)$ is close to $f(c)$
   (B) $\lim\limits_{x\to c} f(x)=f(c)$
   (C) f is defined at c
   (D) $\lim\limits_{x\to c} f(x)$ exists ...

PART III

9. A function f is differentiable at X iff ......
   (A) f is continuous at X
   (B) $\lim\limits_{h\to o} f(X+h)$ exists
   (C) $\lim\limits_{h\to o} \dfrac{f(X+h)-f(X)}{h}$ exists
   (D) The graph of f comes to a sharp point at X ...
10. If $f'(X)=0$ for all X in the domain of f, then
    (A) f is an identify function
    (B) f is a constant function
    (C) f is not defined at X
    (D) f is discontinuous at X ...

APPENDIX I

CONCEPT TRANSFER TEST

Name of the Student :
... ...
Age : ...... Yrs. ...... Months.
Sex : Male/Female
Name of the Institution :
... ...
Class : ...... Section ......
Date : ... ...
Time : From ...... To ......
Duration : .... minutes

Dear Student,

Here is a simple test of your ability to transfer the concepts learnt during experimental teaching. Please answer all questions to the best of your ability. Do not spend undue time on any single item. Your answers are to be written in the space provided on this sheet itself.

N.B. : Supply the missing words or symbols.

PART I

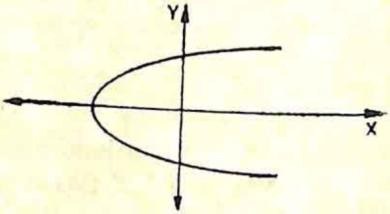
1. The set of all equilateral triangles is a ... ... of the set of all isosceles triangles.

2. If A is the set of all students in the class, then the relation R on AXA defined by

   $R=\{(a, b)/a, \varepsilon$ A and a is as clever as $b\}$

   is ... .

3. Does the curve in the adjoining figure represent graph of a function $Y=f(X)$? ... ...



4. If f maps pupils into their heights and g maps pupils into their weights.

   Then $g\ o\ f$ ... ...

PART II

5. If $f(x)=\left\{\begin{matrix} -1, \text{ if } x < c \\ 1 \text{ if } x < c \end{matrix}\right]$, then

   $\lim\limits_{x \to c} f(x)$... ... ...

6. If $f(x)=\left\{\begin{matrix} 3x-1, & \text{if } x \neq 2 \\ 6 & \text{, if } x=2 \end{matrix}\right]$ then,

   $\lim\limits_{x \to 2} f(x)=$... ... ...

7. $\lim\limits_{x \to c} x=c$. Since for each $\varepsilon>0$, there exists $\delta=$......

   satisfying $\varepsilon-\delta$ definition of limit.

8. If $f(x)=\dfrac{x^2-6x+9}{x-3}$, then f is......at 3

PART III

9. The slope of tangent to the parabola $y=x^2-1$ at $x=1$ is......

10. If $f(x)=\dfrac{1}{x}$, then $f'(x)=$......

APPENDIX J

HEURISTIC TRANSFER TEST

Name of the Student:
... ...
Age:.........Yrs......Months.
Sex: Male/Female
Name of the Institution:
... ...
Class: ....Section............
Date: ......
Time: From.........To...........
Duration:.........

Dear Student,

Here is a simple test of your ability to transfer the heuristics of concepts learnt during experimental teaching. Please answer all questions to the best of your ability. Do not spend undue time on any single item. Your answers are to be written in the space provided for the purpose on this sheet itself.

N.B. Do as directed.

PART I

1. Taking $E=\{0, 1\}$, $A=\{0\}$ and $B=\phi$. Discover relationship between $(A \cup B)^C$ and $A^C \cap B^C$.

Answer :

2. Let A be the collection of all subsets of some set X. Show that '=' is an equivalence relation on A.

Answer :

3. Let $f(X)=\frac{1}{X}$ be the function defined on the set of real numbers. To what value does $f(X)$ approach as X tends to infinity?

Answer :

4. Given $f(X)=X^2$, $X \varepsilon R$, find the approximate vale of $\frac{f(X+h)-f(X)}{h}$ in terms of X, where $h \neq 0$ and h is infinitely small.

Answer :

PART II

5. Obtain inductively, $\lim\limits_{x \to a} \frac{x^n - a^n}{x - a}$, $a > 0$

6. Define $\operatorname{Lim}\limits_{x \to \infty} f(x)$ and show that $\operatorname{Lim}\limits_{x \to \infty} \frac{1}{x} = 0$

7. Given $f(x) = mx + c$, find $\operatorname{Lim}\limits_{h \to o} \frac{f(x+h) - f(x)}{h}$

   Interpret this limit with regard to f.

8. Find the points (if any) at which the following functions are continuous.

   (*i*) $f(x) = \left\{ \begin{array}{l} 1, x \text{ rational} \\ 0, x \text{ irrational} \end{array} \right]$

   (*ii*) $g(x) = \left\{ \begin{array}{l} x, x \text{ rational} \\ 0, x \text{ irrational} \end{array} \right]$

PART III

9. If $y = X^3$, find $\frac{dy}{dx}$ by definition.

10. If the displacement y of the moving particle from the origin at time t is given by $y = -5t^2 + 6t + 10$ find (*i*) initial velocity; and (*ii*) acceleration.

APPENDIX K

CONCEPT RETENTION TEST—1

Name of the Student:... ... ...

Age... ... ...Sex:... ... ...Class:... ... ...Section:... ... ...

Name of the institution:... ... ...

Date:... ... ...Time... ... ...Duration... ... ...

N.B. : Fill up the blanks to complete the following statements.

1. The collection of 'all men of good will' is not a set because... ... ...

2. Inclusion relation ($\subset$) on the set of sets is reflexive and... ... ...but not... ... ...

3. A function f: A$\longrightarrow$B is a rule of correspondence which associates... ... ...element of A to... ... ... element of B.
4. F^{-1} exists iff f is... ... ...and... ... ...
5. $\lim\limits_{x\to 0} f(x)=1$ if $f(x)$ approaches... ... ...as... ... ... approaches c from both sides.
6. $\lim\limits_{h\to 0} \frac{|h|}{h}$ ... ...
7. $\lim\limits_{x\to 1} (5X^2-12X+1)$... ... ...
8. f is continuous at C iff for every $\varepsilon>0$, there exists $\delta>0$ such that... ... ...
9. Derivative of the function f represents the slope of ... ...at point... ... ...
10. $\frac{d}{dx}[X^3-2X^2+5X-10]=$... ... ...

APPENDIX L

CONCEPT RETENTION TEST—2

Name of Student: ... ...
Age:... ... ... Sex:... ... ...Classs... ...Section:... ... ...
Name of the institution:... ... ...
Date:... ... ...Time:... ... ...Duration ...:... ... ...

N.B. : Fill up the blanks to complete the following statements.

1. If ACB, then A$\cup$B=... ... ...
2. If R is a relation from A to B, then RC ... ...
3. If A={1, 2} and B={a, b}, then F={(1, a), (1, b), (2, b)}.........a function from A to B.
4. If f : X$\to$2X, XεR, than $f\,o\,f^{-1}(\;)$=... ... ...
5. $\lim f(X)=1$ iff for every $\varepsilon>0$, there exists $\delta>0$ such that.................
6. $\lim\limits_{x\to 2} \frac{1}{X-2}$ ....
7. $\lim\limits_{x\to 2} \frac{X^2-4}{X-2}$ ....

8. f is continuous at c, iff $\lim\limits_{x \to C} f(X) =$ ....
9. A function f is said to be differentiable at x iff Lim.................exists.
10. Every differentiable function is.................but not *vice-versa.*

APPENDIX M

OBSERVER'S RATING SCALE

Name of the Observer Rater:
Academic Qualification:
Professional Qualification:
Total Teaching Experience:
Present Place of Work:
Occupational Status:
Date and Time of Observation:
Place of Observation:
Class:.................Subject:.................Unit:. . . .

Item	Rating					
A. Method of Induction	0	1	2	3	4	5
B. Introduction of Generalisation	0	1	2	3	4	5
C. Mode of Concept Acquisition Evoked	0	1	2	3	4	5
D. Interjection of Teacher Knowledge	0	1	2	3	4	5
E. Degree of Structure	0	1	2	3	4	5

Place:_______

Date:_______

Signature of the Observer Rater

INSTRUCTIONS TO OBSERVER RATERS

Dear Sir/Madam,

Here is a simple instrument to assess researcher's fidelity to the prescribed teaching models in his doctoral research study.

You are requested to record your free and frank observations by encircling any one number from 0 to 5 against each item of the given rating scale. The rating of 0 would indicate that the item was unobserved or had no occasion to occur. The rating of either 1, 2, 3, 4 or 5 should be assigned in order to indicate the occurrence of teaching behaviour as given below for each item separately.

A. Method of Induction

1. The teacher begins his lesson by presenting positive and negative examples of the concept. He asks pupils to speculate and identify the concept.
2. The teacher presents positive and negative examples. He tells pupils some attributes of the concept and then asks them to identify the concept.
3. The teacher presents positive and negative examples and himself tells pupils the concept involved.
4. The teacher begins his lesson by introducing some general idea at the same level of abstraction.
5. The teacher begins his lesson by presenting a more inclusive concept at the higher level of abstraction and generality (hereinafter referred to as AN ADVANCE ORGANIZER).

B. Introduction of Generalization

1. The teacher elicites formal definition of the concept from the students when all or nearly all of them have identified the concept from examples and non-examples.
2. The teacher himself gives the definition when all or nearly all students have identified the concept from examples and non-examples.
3. The teacher hastens to define the concept after presenting examples and non-examples.
4. The teacher gives definition of the concept before presenting any examples. He does not even illustrate the concept by examples.
5. The teacher does give definition of the concept before presenting any examples, but he illustrates the concept by number of examples.

C. Mode of Concept Acquisition Evoked

1. The teacher asks students to analyse their strategies of concept attainment.
2. The teacher himself analyses the strategies of concept attainment.

3. The teacher does not analyse concept attainment strategies.
4. The teacher relates the concept taught to an 'Advance Organizer' presented earlier.
5. The teacher helps students see the relationship between the concept taught and the 'Advance Organizer'.

D. Interjection of Teacher Knowledge

1. The teacher does not act as primary source of knowledge but shows as if he depends upon the students to attain the concept.
2. The teacher does not act as primary source of knowlege but is inclined to provide cues.
3. The teacher does not hide his knowledge of the subject and is anxious to help students identify the concept.
4. The teacher appears to have read widely but students cannot rely on him for meaningful understanding of the concept.
5. The teacher is the primary source of knowledge and students fully rely on him for meaningful understanding of the concept.

E. Degree of Structure

1. The teacher is liberal. He encourages students to form their own hypotheses using their own strategies even if they are wrong.
2. The teacher is supportive. He cannot avoid suggesting the correct hypotheses and indicating his preference for a particular strategy.
3. The teacher is a task-master. He gets annoyed when wrong answers are given and insists that only the right concept be identified and correct strategy be used.
4. The teacher is authoritarian. He commands the whole affairs in the class. Students are only the followers.
5. The teacher is the controller of the class. He carefully selects material and hems the discussion around it.

S.C.E.R.T., West Bengal Library

ABOUT THE AUTHOR

Dr. U.G.CHITRIV, M.Sc. (Mathematics), Ph. D.(Education) is presently teaching 'Master of Mathematics Education' courses at the Post-Graduate Department of Education, Nagpur University, Nagpur. He has a unique distinction of having taught at all levels from primary to post-graduate which has given him rich experience of and deep insight into educational problems. His basic discipline being mathematics, he devoted himself to the cause of improving mathematical instruction and in the process got involved in the study of 'Models of Teaching'. The present book is the result of his pains-taking work at the Centre of Advanced Study in Education, M.S. University, Baroda where he was a teacher fellow.

Dr. Chitriv has been a meritorious student, a distinguished teacher and an astute researcher throughout his career. He has presented papers on a wide variety of subjects such as 'Analysis of Thinking', 'Cognitive learning Processes', 'Mathematical Creativity' and 'Models of Teaching'. His work on 'Semantic Differential' and 'Cognitive Style' is highly acclaimed. He has been a resource person at the National Workshops on 'Models of Teaching' organized by the NCERT throughout the country and National Seminar on Methodology of Educational Research held at the CASE Baroda. He participated in many national projects including those on 'Development of Training Strategy', Production of Instructional Material for Distance Education' and 'Developing Question Bank on Teaching of Mathematics'.

A few ambitious research scholars are working for their Ph.D. under his guidance.

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